Member's Default Utility Function for Default Fund Design Version 1 ("MDUF v1")

Working Paper

Lead Authors:

David Bell[*], *Estelle Liu*[†], *Dr. Adam Shao*[‡]

Panel Members:

Assoc. Prof. Anthony Asher, Nick Callil, Prof. Geoff Kingston, Dr. David Knox, David Schneider, Tim Unger, Dr. Geoff Warren, Dr. Jack Ding, Wade Matterson, Craig McCulloch, Nicolette Rubinsztein

April 19, 2017

Abstract

The "retirement challenge" represents the need for industry and policymakers to focus on the efficient delivery of retirement income streams. The Government recently announced its support for the development of comprehensive income products for retirement (CIPR) and the industry will likely be shaped towards this direction. This creates significant challenge for industry and policymakers. For instance how do we compare the range of possible outcomes from two competing fund designs? The industry currently lacks clear, quantifiable objectives of what it is trying to achieve for its members. Utility functions have been used in academia for nearly 50 years to address problems of this nature. A working group has spent over a year developing the 'Member's Default Utility Function for Default Fund version 1' (MDUF v1). MDUF v1 is designed to represent a sensible set of preferences for Trustees to assume on behalf of those members whom they have little insight into. This paper details the design considerations investigated by the working group and the final MDUF v1. We illustrate with examples how the utility function can be used to address fund design and policy issues. We note that this paper represents a documentation of research and conversations. Because this is a working paper it may not be perfectly written; however we believe that sharing it adds to the credibility of the research undertaken.

Keywords: utility function, default super fund design, retirement outcomes, retirement income, longevity risk, investment risk, CIPR

[*]David Bell is a lead author of MDUF v1, the Chief Investment Officer at Mine Wealth and Wellbeing and a PhD candidate at UNSW.

[†]Estelle Liu is a lead author of MDUF v1, and a Quantitative Analyst (Retirement Outcomes) at Mine Wealth and Wellbeing.

[‡]Dr. Adam Shao is a lead author of MDUF v1, a Senior Quantitative Analyst (Retirement Outcomes) at Mine Wealth and Wellbeing and an Associate Investigator at CEPAR (ARC Centre of Excellence in Population Ageing Research).

PART I: Background

1 Introduction

Providing retirement outcome solutions is a hugely challenging and complex area. Technically the retirement outcomes problem can be defined as:

"A dynamic, integrated consumption and investment decision problem."

This is a complex problem and there are two important elements to reflect on:

1. An integrated consumption and investment decision based on many factors such as members' characteristics and situation (acknowledging a heterogeneous population) and the large range of available products, solutions and services (yet shortage of product in some key areas). The word "integrated" is an important word to reflect on. At any point in time there is an optimal combined consumption and investment strategy. To determine one in isolation of the other will not result in the best possible solution.

2. A dynamic consumption and investment problem which should be reviewed regularly especially when market conditions and members' situation change.

This problem, combined with the difficulties experienced by the industry in communicating complex financial solutions to fund members who on average have low levels of financial literacy, creates an even greater challenge for the industry.

The industry lacks a clear objective. The proposed objective of superannuation set in the Superannuation (Objective) Bill 2016 is *"To provide income in retirement to substitute or supplement the Age Pension"*. However such an objective remains aspirational rather than precise. Without a precise definition it is not possible to undertake the quantitative work necessary to address the issue of retirement solution design.

While there are existing retirement outcome metrics they are generally flawed and fail to adequately represent the preferences of a default member. We created a working group of academics and industry professionals to determine an appropriate and sensible set of objectives for trustees to assume on behalf of their default fund members, members they know little about. We then converted these preferences into a mathematical function, known as a utility function. This function has been given the name "Member's Default Utility Function Version 1" or "MDUF v1" for short.

In Part I of the paper we provide the background knowledge of the superannuation system and utility theory. The development of the MDUF v1 is detailed in Part II of the paper.

1.1 Objectives of superannuation system

The Government recently agreed on the recommendations made by the Financial System Inquiry (FSI) to enshrine the objective of the superannuation system in legislation. This is aiming to provide guidance to the industry for a better superannuation system with considerations around fairness, adequacy and dignity. Different parties such as the Treasury, the Association of Superannuation Funds of Australia (ASFA), and the Australian Institute of Superannuation Trustees (AIST) stated their objectives for the Australian superannuation system in slightly different ways. However, it is clear that the objectives are all around sustainable retirement income.

- **The treasury adopted FSI's view:** provide income in retirement to substitute or supplement the Age Pension.

- **ASFA's view:** help achieve the best retirement outcomes for members of super funds through the development of good public policy and industry best practice.
- **AIST's view:** provide equitable, adequate and sustainable retirement incomes by considering the relationship with the Age Pension and the health care system.
- **Super system review:** enhance retirement incomes with a whole of life focus and include a retirement income stream product chosen by the trustee to address longevity, inflation and investment risks.

1.2 Evolution of superannuation system

The superannuation industry is changing its focus in a number of different areas highlighted below:

- Shift of focus from lump sum benefit to retirement income stream
- Shift of focus from managing investment risk and return to managing distribution of retirement outcomes
- In addition to investment risk, other risk factors such as mortality risk, longevity risk, liquidity risk, sequencing risk and inflation risk are also key risks to manage for retirement.
- As such the universe of retirement solutions broadens to include:
  - Investment solutions: investing in different asset classes and or pre-mixed diversified options offered by trustees or investing through Self Managed Super Fund (SMSF) whichever meet members' investment needs.
  - Mortality/Longevity solutions: purchasing products that provide longevity protection to reduce the risk of members outliving their savings before death. Possible products can be life annuities and longevity pooling products which are currently offered in Australia.
  - Default retirement solutions: could be a mix of investment and mortality/longevity solutions mentioned above recommended based on broad assumptions for members. The Government announced its support to the development of comprehensive income products for retirement (CIPRs) and will facilitate trustees pre-selecting these products for members.
  - Advised retirement solutions: getting tailored advice from financial advisers for appropriate income drawdowns in retirement based on personal situation.
  - Personal retirement solutions: in between default and advised retirement solutions where known personal information is used for designing better retirement solutions. An example could be a digital advice service.

To manage retirement outcome solutions for members, we need guidance as to how we should trade off the level of retirement income against its certainty (certainty that the member will receive the nominated income stream each year suggested by the trustee) and its variability (the range of possible outcomes due to variability in investment return and mortality outcomes etc.). We should also consider the scenarios around residual benefits and whether these are valued by members. To do this, we need to shift our focus from pure financial outcomes to the

level of satisfaction associated with the range of possible outcomes; we can measure this level of satisfaction by using utility functions.

1.3 Designing retirement income strategies

Retirement income can be measured in absolute dollar amounts, by replacement rates[1] or as a comparison to certain living standards. There are several ways of quantitatively designing retirement income strategies based on retirement outcome projection techniques:

- Seek to maximise the expected retirement outcome level measured in either absolute dollar amount or by replacement rates. In this case, we do not consider setting a target for income and the year by year fluctuation of the retirement outcome is not managed. This leaves us open to the risk of not managing for the risk of unacceptable outcomes.

- Determine an appropriate target and seek to minimise the expected failure rate with respect to achieving that target. For example, measuring the number of years that a member will experience income shortfall below a chosen target. There are different ways of choosing the appropriate target. For example it could be achieving 70% replacement rate for retirement or achieving ASFA modest or comfortable living standards. An issue with this approach is that we ignore the consideration of how far we may fall below the target in some scenarios. In this case, we do consider setting a certain target for income but the year by year fluctuation of the retirement outcome is still not managed.

- Seek to maximise the expected utility of retirement outcomes. A utility function is used to reflect the preferences of individuals, and it effectively puts values ('scores') on the retirement experience of a set of cash flows in retirement. A utility function places higher values on favourable outcomes while marking down poor outcomes such as a consistently lower level of retirement income or a one-off hit. Unlike the previous two methods, the utility function approach is capable of balancing different aspects of retirement outcomes and allowing for subjective adjustments to how the members value different outcomes. For example, bequests may be considered as an important aspect in retirement solutions. A utility function can be designed to accommodate these outcomes.

2 Background on utility theory

2.1 What is a utility function?

Utility functions are mathematical functions that are designed to reflect the preferences of an individual. They can reduce the dimension of problems to something that is easier to measure and compare through ranking alternatives according to their utility outcomes. They can be very useful in objectively addressing important questions such as comparing investment opportunities, constructing portfolios, designing and comparing products and assessing the value of advice. A higher utility outcome is always preferred over a lower utility outcome. In theory, economically rational investors make choices consistent with maximising the expected value of the utility

[1] **Replacement rate:** the proportion of a member's current employment income that the current value of the member's superannuation benefits would represent if paid to the member as an income stream over the member's retirement.A commonly expressed aspiration for adequacy is for a superannuation balance large enough to provide an income stream (including capital drawdown) of around $60 - 70\%$ of pre-retirement income over a 25-30 year period.

function when considering a range of possible utility outcomes accounting for risk factors such as investment returns and mortality experience. Utility can be defined over either wealth or consumption over time. However it can also be defined over a bundle of items. The use of a utility function involves a number of consumers' behavioural assumptions such as their risk preference, their reaction to changes in economic factors and their wealth level.

In a perfect world we would construct individual retirement plans for each member with a perfect understanding of their preferences. There are two significant hurdles we face in practice. The first one is the operational restrictions and costs which mean that members are commonly aggregated into pooled solutions; we therefore need to make broad assumptions of an 'average' member which might not truly reflect individual preferences. The second one is that academic literature suggests that even if the administrative restrictions did not exist, it is still very hard to accurately elicit the preferences of individuals. One example could be the basic processes for risk profiling applied by some parts of the financial planning industry. General questionnaires on risk preferences are put forward to clients but these results do not convert well into an economic measure of risk aversion.

Utility functions can take many forms. Widely used utility functions take mathematical forms such as power functions, log functions, linear functions, etc. with respect to wealth or consumption. For a default fund we seek a sensible paternal utility function appropriate for a trustee to assume on behalf of its membership. This could then be considered by all super funds in the design of their default funds. However they could 'opt-out' and develop their own version of a utility function if they believe they have greater insight into the preferences of the members in their fund. We suspect two types of 'opt-out' from other super funds:

- Homogeneous opt-out: a super fund believes their members possess very different characteristics in risk aversion, bequest etc. compared to those presented in MDUF v1.
- Heterogeneous opt-out: a super fund believes the heterogeneity of their members' characteristics cannot be represented by one single utility function. This would be the case if all members received financial advice and preference assessments.

In both cases, the concept and framework of utility maximisation remains highly relevant. Once established, a utility function becomes our objective beacon for guiding our forward looking decisions around the design of retirement solutions. It can be an excellent ex-ante guide for us to recommend retirement strategies over time. There are a number of aspects to be considered in designing a utility function:

- The utility function is designed from the trustee's perspective and as a result it reflects existing regulations and the spirit of proposed regulations.
- We assume no unique insights into members beyond their age, gender, balance and contribution levels.
- A trustee is obliged to make (sometimes paternal in nature) decisions that they believe is in the best interests of members. One output of this is default fund design. A member always has the right to opt out of the default.
- The utility function needs to achieve its purpose with minimal complexity and it should be broadly understandable by an appropriately well resourced super fund, possibly with guidance from a consultant.
- Highly debatable assumptions used in constructing the utility function need to be avoided.

Given all of this we name the function the "Member's Default Utility Function v1" (MDUF

v1). The v1 acknowledges that we believe this function should be reviewed periodically as the industry continues to evolve, particularly as new research into members' preferences become available.

2.2 Is the expected utility theory appropriate for the design of default funds?

The expected utility framework provides decision criteria for variable outcomes. The two big risks to retirement outcomes, which super funds can address, are variability in investment returns and mortality outcomes. Given the range of possible investment outcomes and mortality outcomes, the resulted range of utility outcomes can be calculated. The expected utility is then the sum of each utility outcome weighted by the likelihood of these outcomes occurring. If a simulation technique is used to generate a large enough range of risk scenarios, then the expected utility can be estimated as the average of all the utility outcomes. Figure 2.1 illustrates the calculation of expected utility for a single cashflow problem.

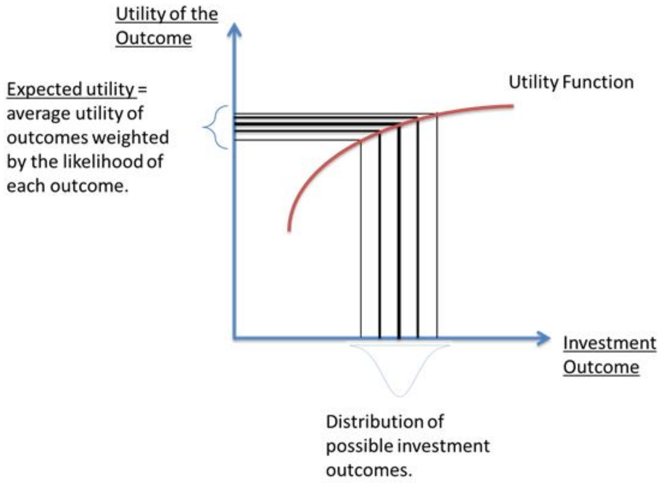


Figure 2.1: Illustration of expected utility calculation for a single cashflow problem

The expected utility theory is based on four axioms which define a rational decision maker. They are:

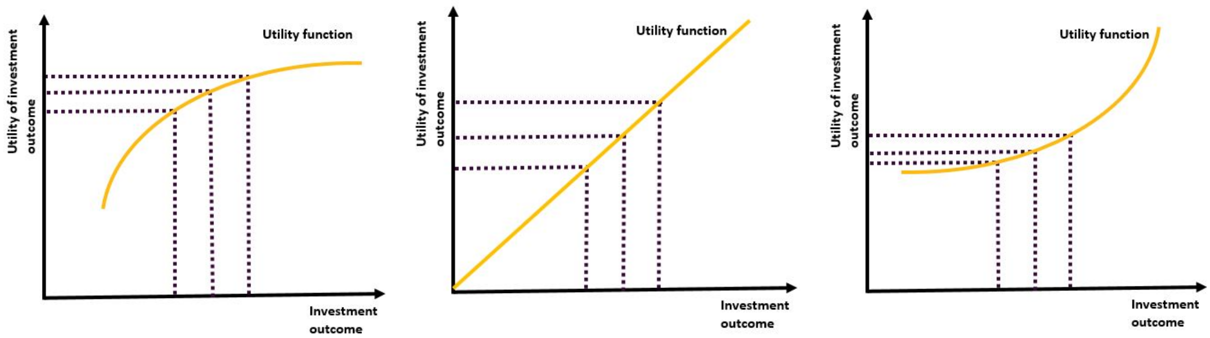
- Completeness: an individual can always decide between two alternatives.
- Transitivity: an individual can decide consistently
- Independence: two gambles mixed with a third one maintain the same preference order as when the two are presented independently of the third one.
- Continuity: if an individual has clear preferences over three gambles, then there should be a possible combination of the most and the least preferred options in which the individual is then indifferent between the mix and the one which sits in the middle.

Expected utility theory is only appropriate when these four axioms hold and the individual is considered to be a rational decision maker under this definition. However, there have been some studies such as Starmer (2000) that revealed empirical evidence of the violation of these axioms by individual decision makers. The question is then whether it is still appropriate for us to use the expected utility theory to design the default funds for our members if not all of them are rational decision makers? We believe the answer is "yes" and this is because we assume the role

of a trustee in providing the default retirement products is to act on behalf of their members' best interest throughout the design process. One important ramification of this assumption is that the trustee will be making a 'rational' decision on behalf of individual members who may not always make rational decisions themselves.

2.3 Types of utility function based on risk preferences of investors

Assume $u(w)$ is the mathematical formulae representing the utility function of an individual with wealth (outcome) equal to w. An individual can be identified as risk averse, risk neutral or risk loving depending on the shape of their utility curve. Table 2.1 explains what each of them means in detail.



(a) Risk averse (b) Risk neutral (c) Risk loving

Figure 2.2: Types of risk preference of investors

Table 2.1: Comparing types of risk preference of investors

Risk averse	Risk neutral	Risk loving
An investor prefers a low level of risk on his investments. They will accept a higher level of risk only if the increase in expected returns is sufficient.	An investor is indifferent between choices with equal expected payoffs regardless of the riskiness of each choice	An investor seeks higher risk investments. They will only accept lower risk if it is accompanied by a sufficient improvement in expected returns.
Concave utility curve	Linear utility curve	Convex utility curve
Marginal utility is decreasing as wealth is growing.	Marginal utility is unchanged as wealth is growing.	Marginal utility is increasing as wealth is growing.
Marginal utility is lower for higher outcome.	Marginal utility is constant for all outcome levels.	Marginal utility is higher for higher outcome.

Over the long term, lower expected returns are usually associated with lower risk investments and higher expected returns are associated with higher risk investments. As most investors expect to be compensated for taking on additional risk, they are considered to be risk averse investors.

Risk aversion captures an individual's conservativeness towards risk. The more risk averse an individual is, the more pronounced is the utility loss resulting from a reduction in outcome than the utility gain from an increase in outcome of the same amount, i.e. the higher the risk aversion parameter, people are more afraid of loss.

The classic mean-variance optimisation framework is useful for understanding the behaviour of investors with different risk preferences. In the mean-variance construct investors seek to maximise expected return for a selected level of risk or minimise risk for a targeted expected return. A straight line starting at the risk-free rate tangent to the Efficient Frontier is known as the Capital Market Line (CML). The point on the Efficient Frontier tangent to the CML is known as the Market Portfolio (M). The theory (Markowitz, 1952 and Tobin, 1958) is that all investors should invest along the CML, a portfolio combining the market portfolio (M) and cash (either invested or borrowed).

In a utility maximising framework we can make the following observations, relative to a starting point of the Market Portfolio (M):

- All investors would actively seek a portfolio which would move them north of M.
- A risk neutral investor is happy to move east of M as they are indifferent to risk. However, in moving east of M they then have an opportunity to move north as well so they would invest on the north-east end of the CML.
- A risk averse investor would be happy to move directly west of M. But a portfolio with such a profile is theoretically unachievable. They may consider a move south-west of M if the trade-off between lower expected returns and reduced risk was appropriate. A risk averse investor may seek to move east if the additional return available from moving north up to the CML is viewed as sufficient compensation for the extra risk. A risk averse investor will sit on the CML but where they sit will depend on their own risk preference and the slope of the CML (the Sharpe Ratio of M).
- A risk loving investor would be happy to move directly east of M. They may also be prepared to accept a lower expected return in return for higher risk (i.e. head south-east), but in heading in this direction they would always take the opportunity to subsequently move north and return to the CML. A risk loving investor may consider moving west of M; however they would prefer higher risk, and so to move west they would seek to be compensated with a higher expected return. However moving north-west of M is theoretically not possible. A risk loving investor would sit at the north-east end of the CML.
- All investors should sit on the CML. A risk averse investor would sit south-west relative to the risk neutral and risk loving investors.

The mean-variance framework implicitly assumes that portfolio returns are normally distributed. However we know that portfolios can be constructed with different shaped return distributions (e.g. through the use of option strategies) and that the Age Pension reflects a floor on outcomes. Individuals with different risk preferences have a preference for different outcome distributions. This is demonstrated from a distributional skew perspective in Figure 2.4.

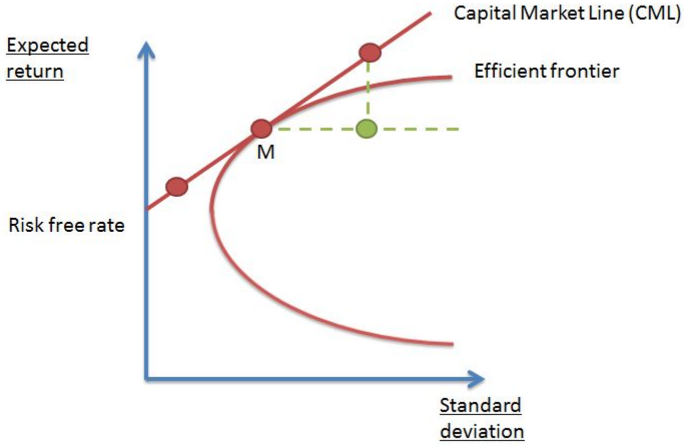
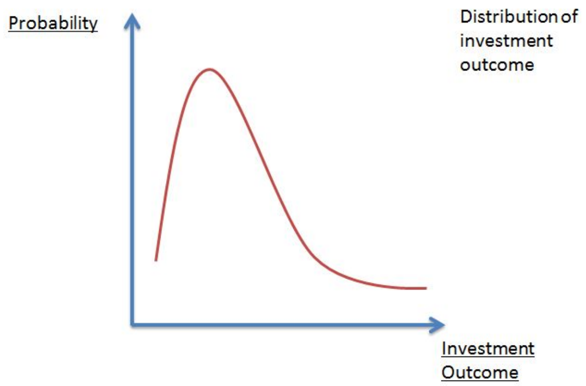
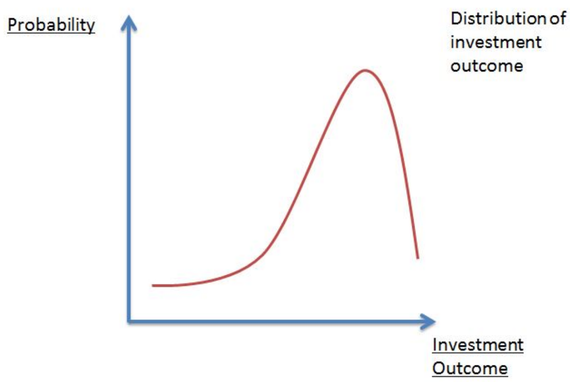


Figure 2.3: Mean-variance portfolio theory and risk preference of investors



(a) Risk averse



(b) Risk loving

Figure 2.4: Preferred distributions for investment outcome based on risk preference of investors

3 Classes of utility function

As most investors are considered to be risk averse we will focus on analysing different classes of utility function in this section for risk averse investors.

3.1 Only focus on terminal wealth (w)

There is a lot of literature which discusses investment strategies in the accumulation phase of defined contribution pension schemes using expected utility maximization of terminal wealth. For example, Boulier et al. (2001), Haberman and Vigna (2002), Deelstra et al. (2003), Devolder et al. (2003), Battocchio and Menoncin (2004),Cairns et al. (2006), Xiao et al. (2007), Gao (2008), and Di Giacinto et al. (2011). This is based on the simplest form of utility function as the input is only one number (the terminal wealth). Risk aversion of an investor can be analysed from two different perspectives:

- Absolute risk aversion (ARA): measures the individual's risk aversion level as the absolute dollar value of their wealth changes. Absolute risk aversion is measured by the negative

of the ratio between the second derivatives and the first derivatives of the utility function with respect to wealth as shown below:

$$ARA(w) = -\frac{u''(w)}{u'(w)} \tag{3.1}$$

- Relative risk aversion (RRA): measures the individual's risk aversion level with respect to percentage changes in their wealth. Relative risk aversion is measured by the product of the absolute risk aversion value and the investors' wealth as shown below:

$$RRA(w) = -w \times \frac{u''(w)}{u'(w)} = w \times ARA(w) \tag{3.2}$$

Table 3.1 explains the different types of risk aversions by considering the different attitudes of investors towards investing in risky assets when their wealth changes. Absolute risk aversion focuses on dollar amount changes in risky investment and relative risk aversion focuses on the percentage changes of the portfolio invested in risky assets.

Table 3.1: Types of risk aversions

	Increasing (I)	Constant (C)	Decreasing (D)
Absolute risk aversion (ARA)	IARA: the investor tends to increase the dollar amount invested in risky assets when his/her wealth increases	CARA: the investor tends not to change the dollar amount invested in risky assets when his/her wealth increases	DARA: the investor tends to decrease the dollar amount invested in risky assets when his/her wealth increases
Relative risk aversion (RRA)	IRRA: the investor tends to increase the percentage of his/her wealth invested in risky assets when his/her wealth increases	CRRA: the investor tends not to change the percentage of his/her wealth invested in risky assets when his/her wealth increases	DRRA: the investor tends to decrease the percentage of his/her wealth invested in risky assets when his/her wealth increases

The literature tends to most commonly consider the cases of constant risk aversion (i.e. CARA or CRRA).

3.1.1 Comparison of ARA(w) and RRA(w)

For investors who are CARA, as their wealth increases, the dollar amount they invest in risky assets would not change. This means as a percentage of their wealth, their investment in risky assets actually decreases over time. As a result, CARA investors are also DRRA investors but the inverse is not always true. Similarly, for investors who are CRRA, as their wealth increases, the percentage of their wealth invested in risky assets does not change. This means the dollar amount invested in risky assets actually increases over time. As a result, CRRA investors are also IARA investors but the inverse is also not always true.

3.1.2 Hyperbolic absolute risk aversion (HARA)

Almost all applied theory and empirical work in finance uses some member of hyperbolic absolute risk aversion (HARA) (also known as linear risk tolerance (LRT)) utility functions.

$$u(w) = a\Big(\frac{\rho_w}{1-\rho_w}\Big)\Big(\frac{w}{\rho_w} - \hat{w}\Big) + b \tag{3.3}$$

where ρ_w is the coefficient of relative risk aversion for the HARA utility function defined over wealth (w), a and b are constants as a result of linear transformation. HARA specializes to a number of important utility functions including exponential utility, power utility, log utility, and quadratic utility.

- **CARA or Exponential utility:**

$$u(w) = -e^{-\lambda_w w} \tag{3.4}$$

This is a special case of HARA with $\hat{w} = \Big(\frac{1-\rho_w}{\rho_w}\Big)\frac{1}{\lambda_w}$, $a = \Big(\frac{-\rho_w}{1-\rho_w}\Big)^{-\rho_w}\lambda_w^{1-\rho_w}$, $b = 0$. This gives the following expression of $u(w)$.

$$u(w) = \Big(1 - \frac{\lambda_w w}{1-\rho_w}\Big)^{1-\rho_w} \tag{3.5}$$

$$\lim_{\rho_w \to \infty} u(w) = \lim_{\rho_w \to \infty} \Big(1 - \frac{\lambda_w w}{1-\rho_w}\Big)^{1-\rho_w} = -e^{-\lambda_w w} \tag{3.6}$$

$$ARA(w) = -\frac{u''(w)}{u'(w)} = -\frac{-\lambda_w^2 e^{-\lambda_w w}}{\lambda_w e^{-\lambda_w w}} = \lambda_w \tag{3.7}$$

As $ARA(w) = \lambda_w$ is a constant value and independent of w, we call this type of utility function constant absolute risk aversion (CARA). λ_w is the coefficient of absolute risk aversion. For investors with CARA utility function, when their wealth increases, they will not change the absolute dollar amount invested in risky assets.

- **CRRA:**

$$u(w) = \begin{cases} \dfrac{w^{1-\rho_w}}{1-\rho_w}, \text{ if } \rho_w > 0,\ \rho_w \neq 1 \text{ (This is known as power utility)} \\ \ln w, \text{ if } \rho_w = 1 \text{ (This is known as log utility, a special case of power utility)} \end{cases} \tag{3.8}$$

CRRA is a special case of HARA with $\hat{w} = 0$, $a = \rho_w^{-\rho_w}$. In the case of power utility, $b = 0$.

$$RRA(w) = -w \times \frac{u''(w)}{u'(w)} = -w \times \frac{-\rho_w(1-\rho_w)w^{-1-\rho_w}}{(1-\rho_w)w^{-\rho_w}} = \rho_w \tag{3.9}$$

As $RRA(w) = \rho_w$ is calculated to be a constant value and independent of w, we call this type of utility function constant relative risk aversion (CRRA). ρ_w is the coefficient of relative risk aversion. For investors with CRRA utility function, when their wealth increases, they will keep a constant proportion of their wealth invested in risky assets. In the case of log utility, $b = \frac{1}{\rho_w - 1}$. This gives the following expression of $u(w)$

$$u(w) = \frac{w^{1-\rho_w} - 1}{1-\rho_w} \tag{3.10}$$

If we replace $1 - \rho_w = \varphi$ then

$$\lim_{\rho_w \to 1} u(w) = \lim_{\varphi \to 0} \frac{w^{\varphi} - 1}{\varphi} = \lim_{\varphi \to 0} \frac{\frac{d}{d\varphi}(w^{\varphi} - 1)}{\frac{d}{d\varphi}\varphi} = \lim_{\varphi \to 0} \frac{w^{\varphi} \ln w}{1} = \ln w \tag{3.11}$$

$$RRA(w) = -w \times \frac{u''(w)}{u'(w)} = -w \times \frac{\frac{1}{-w^2}}{\frac{1}{w}} = 1 \tag{3.12}$$

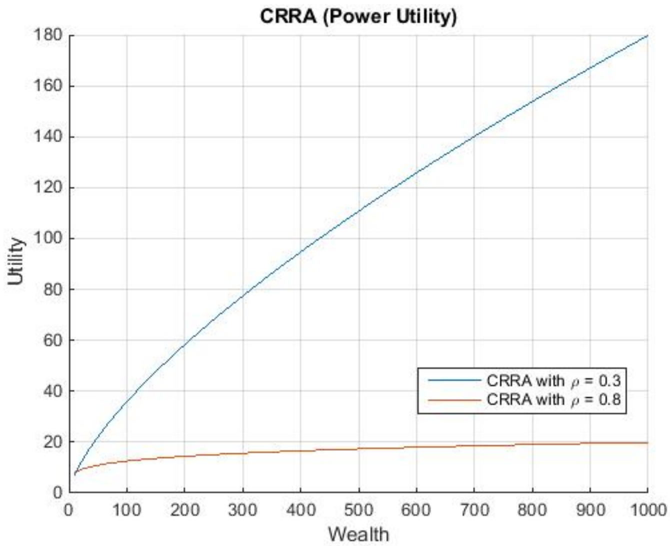


Figure 3.1: Degree of risk aversion and the value of ρ_w

The value of ρ_w determines the degree of risk aversion. This is shown in figure 3.1, where we plot two power utility functions with $\rho_w = 0.3$ and 0.8. A higher ρ_w means a higher degree of risk aversion as the marginal utility from gaining additional wealth is lower.

- **Quadratic utility:** Quadratic utility is a special case of HARA with $\hat{w} < 0$, $\rho_w = -1$, $a = 1$, $b = 0$ and $w < -\hat{w}$. This gives the following expression of $u(w)$.

$$u(w) = -\frac{1}{2}(-w - \hat{w})^2 \tag{3.13}$$

Quadratic utility is neither CARA nor CRRA. To classify quadratic utility function, we derive the equations for ARA and RRA as below.

$$ARA(w) = -\frac{u''(w)}{u'(w)} = \frac{1}{-w - \hat{w}} \tag{3.14}$$

$$RRA(w) = -w \times \frac{u''(w)}{u'(w)} = \frac{w}{-w - \hat{w}} \tag{3.15}$$

The results tell quadratic utility function is IARA and DRRA. Economists avoid using quadratic utility because IARA and DRRA have unrealistic behavioural implications.

3.1.3 Comparing CARA and CRRA

Utility functions with constant risk aversion, i.e. CARA or CRRA, are the most commonly used in academic literature on maximization of expected utility of terminal wealth in DC accumulation phase. Some papers that discuss about using CARA utility function include Devolder et al. (2003) and Battocchio and Menoncin (2004). There are a lot of papers which discuss CRRA as the most widely used utility function especially when it is defined over life-cycle consumption. These include Tobin and Dolde (1971), Mehra and Prescott (1985), Gourinchas and Parker (2002), Chetty (2006) , Schechter (2007), Yogo (2009), Ameriks et al. (2011), and Lockwood (2014).

It is believed that CRRA describes more logical investors' behaviour than CARA which is consistent with a lot of economics and financial literature. Consider a scenario where an investor has $\$10,000$ and currently invests $\$5,000$ (50%) in risky assets. CARA describes when an investor grows their wealth from $\$10,000$ to $\$1,000,000$, they will still invest only $\$5,000$ in risky assets. However, under CRRA they will always invest 50% which is now $\$500,000$ of their wealth in risky assets.

We prefer CRRA over CARA for the purpose of this paper given the more realistic assumption on investors' behaviour by CRRA. However, there is also a concern about the implications of CRRA particularly as wealth levels become very high. If we continue the above example, it may be realistic to consider that an investor with $\$1,000,000$ would allocate $\$500,000$ to risky assets. But if an investor has \$1 billion, would it be realistic to assume that they would invest \$500 million in risky assets? Generally there is a concern that the CRRA assumption does not account for a limit to how much absolute financial risk an individual is prepared to take.[2]

We feel that the issues of CRRA in dealing with extremely high wealth levels should not be a limitation for the application under our context. Members in a default fund with extremely high balances are really rare especially with the new \$1.6 million transferable pension balance cap proposed in Budget 2016.

3.2 Lifetime consumption (c)

In the history of Australian defined contribution scheme, the industry focus has generally been on terminal wealth, namely the lump sum benefit available at retirement. However, the definition of superannuation and the consideration of managing longevity risk guides us to focus on retirement income. As a result, we want to discuss about lifetime consumption problems in addition to terminal wealth.

Lifetime consumption problems take into account multi-period consumption and are based on the expected utility theory. Further exploration is required to determine whether time-separable preference or non-time-separable preference is more appropriate for our context.

Compared to the utility functions which only focus on terminal wealth, ones that account for lifetime consumption are more complicated as they take into account streams of consumption over time.

There are two additional components to consider in a multi-period utility function compared to a single period one. They are the time preference discount factor (also referred to as a patience parameter) and the intertemporal elasticity of substitution.

[2] Interestingly research such as Carroll (2000) provides alternative evidence that the rich actually hold higher risk and more concentrated portfolios.

- **Time preference discount factor:**
  Time preference discount factor considers time value of money. It can be objective if price inflation or wage inflation is used as the discount factor to allow for comparision of consumption in today's dollar. It can also be subjective if the utility function takes into account personal preferences and an individual's patience to wait and consume in the future.

  Wage inflation might be more suitable as a discount factor compared to price inflation due to the fact that wage inflation takes into account increases in living standards in addition to the cost of consumer goods.

- **Elasticity of Intertemporal Substitution (EIS):**
  Elasticity of Intertemporal Substitution (EIS) considers more than time value of money, rather it focuses on real return of investments. Basically, it suggests that if market conditions change and investors form different perceptions of the future market return (real), then they will change their preference with regard to deferring consumption into the future. If they expect better future real return, then they might prefer to consume less today to leave a bit more money in the investment to grow for future consumption. The parameter of EIS determines the sensitivity of the investors' consumption behaviour towards changes in their expectations of the market real return.

  Historical research has shown the value of EIS to be relatively low. i.e. investors' consumption behaviour is insensitive towards changes in expectation for future market performance. In addition, it might be hard to incorporate EIS explicitly in practice because this would involve a much more complex multi-period utility function which is non-time-additive. This would make the whole model less tractable. We illustrate how EIS interact with the risk aversion parameter in section 3.2.1.

3.2.1 Two-period Problem

The concept of utility function with lifetime consumption can be illustrated using a two period example as shown below.

$$U(c_t, c_{t+1}) = u(c_t) + \beta u(c_{t+1}) \tag{3.16}$$

$$V_t = \mathbb{E}_t[U(c_t, c_{t+1})] = u(c_t) + \beta \mathbb{E}_t[u(c_{t+1})] \tag{3.17}$$

$$\max_{c_t, c_{t+1}} V_t = \max_{c_t, c_{t+1}} \{u(c_t) + \beta \mathbb{E}_t[u(c_{t+1})]\} \tag{3.18}$$

where c_t is the consumption at time t. $u(c_t)$ represents the utility of consumption at time t. $U(c_t, c_{t+1})$ is the total utility of consumption over two periods. $\mathbb{E}_t[\cdot]$ is the mathematical expression for calculating the expected value at time t. In this paper, we use V_t to express the expected utility at time t of future consumptions. The total utility depends on how much the investors consume today at time t and on how much they consume in the future time $t+1$. The parameter β is the time preference discount factor, it captures the weight that investors place on the future relative to today. If $\beta = 1$, then they treat consumptions today and in the future equally. Alternatively, if $\beta < 1$ then they value today's consumption more than the future. This is intuitive and β can be the discount rate of interest representing the time value of money or a subjective discount rate perceived by the investors themselves. Given the role of a trustee, the chosen parameters should reflect objective views to represent the broad membership. $\beta = 1$ is a realistic assumption if we are comparing consumption in real terms. This means inflation rate (price inflation or wage inflation) can be suitable time preference discount factors in this case.

To illustrate the idea of a two-period consumption problem, consider the following scenario. An

investor has 2 hundred and is thinking how to spend it over two years so he is most satisfied with the outcome (utility). Assume the investor uses CRRA utility function with $\rho_c = 0.3$ and $\beta = \frac{1}{1+2.5\%}$ discounting for price inflation. ρ_c is the coefficient of relative risk aversion of CRRA utiltiy function defined over consumption. Table 3.2 shows the results of this illustration. In the table the investor's utility is highest when he splits the 2 hundred to spend 1 hundred each year. Note that in theory the optimal split would be 104 in year 1 and 96 in year 2. Also note that because $\beta \neq 1$ that the utilities derived under scenario 2 ($c_1 = 80$, $c_2 = 120$) and scenario 4 ($c_1 = 120$, $c_2 = 80$) are not equal.

Table 3.2: Two-period consumption problem illustration

c_1	c_2	$u(c_1)$	$u(c_2)$	$U(c_1, c_2)$	Rank
50	150	20.66	46.23	65.77	4
80	120	29.27	39.34	67.65	3
100	100	34.46	34.46	68.07	1
120	80	39.34	29.27	67.89	2

- **Illustration of EIS:**
For consumption problems involving more than 1 period, the elasticity of intertemporal substitution (EIS) needs to be considered too. EIS measures the change in consumption growth in response to change in expected investment return. We can express EIS in the mathematical formulae below:

$$EIS = \frac{d\ln(\frac{c_{t+1}}{c_t})}{d\ln(1+R)} \tag{3.19}$$

where $c_{t+1} = (w_t - c_t)(1 + R)$. If we look at power utility as an example for a 2-period problem, then:

$$\max_{c_t, c_{t+1}} \frac{c_t^{1-\rho_c}}{1-\rho_c} + \beta\mathbb{E}_t\left[\frac{c_{t+1}^{1-\rho_c}}{1-\rho_c}\right] \tag{3.20}$$

Assume R is predetermined, then $\mathbb{E}_t\left[\frac{c_{t+1}^{1-\rho_c}}{1-\rho_c}\right] = \frac{[(w_t-c_t)(1+R)]^{1-\rho_c}}{1-\rho_c}$.

$$\frac{\partial}{\partial c_t}\left\{\frac{c_t^{1-\rho_c}}{1-\rho_c} + \beta\mathbb{E}_t\left[\frac{c_{t+1}^{1-\rho_c}}{1-\rho_c}\right]\right\} \tag{3.21}$$

$$=\frac{\partial}{\partial c_t}\left\{\frac{c_t^{1-\rho_c}}{1-\rho_c} + \beta\frac{[(w_t-c_t)(1+R)]^{1-\rho_c}}{1-\rho_c}\right\} \tag{3.22}$$

$$=c_t^{-\rho_c} - \beta(1+R)c_{t+1}^{-\rho_c} \tag{3.23}$$

$$=0 \tag{3.24}$$

$$\ln\left(\frac{c_{t+1}}{c_t}\right) = \frac{1}{\rho_c}\ln\beta + \frac{1}{\rho_c}\ln(1+R) \tag{3.25}$$

$$EIS = \frac{d\ln(\frac{c_{t+1}}{c_t})}{d\ln(1+R)} = \frac{1}{\rho_c} \tag{3.26}$$

The result shows that for power utility, EIS for consumption is equal to the inverse of $RRA = \rho_c$. For other utility functions, EIS can be complex to solve.

3.2.2 Multi-period Problem

It is easy to generalise the expression for lifetime utility with multi-period consumption if we assume the investors have a time additive utility function of the form below. Non-time-separable utility function will be briefly discussed in section 3.2.4.

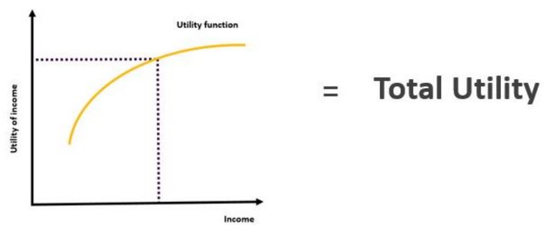
$$U(c_t, c_{t+1}, \ldots, c_T) = \sum_{\tau=t}^{T} \beta^{\tau-t} u(c_\tau) \tag{3.27}$$

$$V_t = \mathbb{E}_t[U(c_t, c_{t+1}, \ldots, c_T)] = \mathbb{E}_t\left[\sum_{\tau=t}^{T} \beta^{\tau-t} u(c_\tau)\right] \tag{3.28}$$

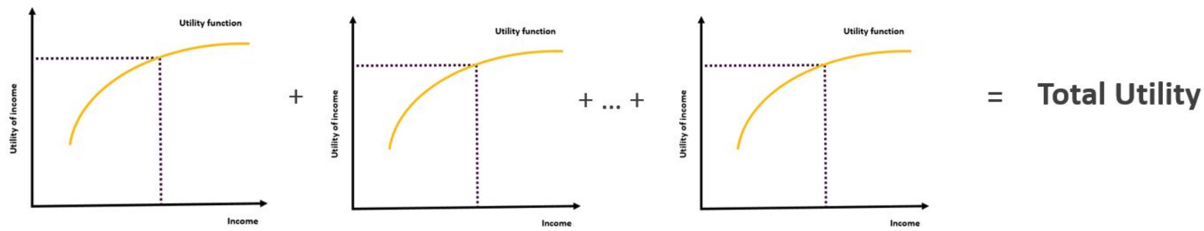
$$\max_{c_t, c_{t+1}, \ldots, c_T} V_t = \max_{c_t, c_{t+1}, \ldots, c_T} = \mathbb{E}_t\left[\sum_{\tau=t}^{T} \beta^{\tau-t} u(c_\tau)\right] \tag{3.29}$$

Figure 3.2 illustrates how multi-period utility outcome is calculated by the summation of a series of single-period utility outcomes.

Figure 3.2: Utility of multi-period cash flows



(a) Utility of a single cash flow



(b) Utility of a collection of cash flows

3.2.3 Specification in addition to wealth and consumption

It is considered that in addition to consumption, people might place value on other aspects such as bequest motive and leisure. In this case, there will be additional terms to be included in the utility function.

Bequest motive:

The following shows an example of the time-separable power utility at time zero discussed in Cocco, et al. (2005) that considers lifetime consumptions and bequest motive. We redefine the utility function at time t in the form below:

$$U(c_t, c_{t+1}, \ldots, c_T) = \sum_{i=0}^{T-t} \beta^i \Big\{ {}_ip_t \; u(c_{t+i}) + {}_{i-1|}q_t \; v(b_{t+i}) \Big\}, \tag{3.30}$$

where

$$u(c_t) = \frac{c_t^{1-\rho_c}}{1-\rho_c}$$

and

$$v(b_t) = k\frac{b_t^{1-\rho_c}}{1-\rho_c}.$$

$\rho_c > 0$ measures the level of risk aversion in the sense that as ρ_c increases the person indeed exhibits a more risk averse behaviour. b_t is the level of bequest at time t if the person dies between $t-1$ and t. k is a measure of the strength of bequest motive. $k > 0$ indicates the person will always keep some liquid assets to bequest. ${}_ip_t$ is the probability of being alive at time $\tau + k$ conditional on being alive at time t. ${}_{i-1|}q_t$ is the probability of dying between time $t+i-1$ and $t+i$ conditional on being alive at time t.

At time t, the objective is to maximise the expected present value of utilities through the lifetime. This is expressed in the following equation:

$$V_t = \max_{c_t, c_{t+1}, \ldots, c_T} \mathbb{E}_t \left[\sum_{i=0}^{T-t} \beta^i \left\{ {}_ip_t \ u(c_{t+i}) + {}_{i-1|}q_t \ v(b_{t+i}) \right\} \right] \tag{3.31}$$

where $\mathbb{E}_t$ is the expectation operator.

Lockwood (2014) separates the consideration of risk aversion over consumption and bequest by including a threshold consumption level in the bequest motive function. The bequest motive function is:

$$v(b_t) = \left(\frac{\phi}{1-\phi}\right)^{\rho_c} \frac{\left(\frac{\phi}{1-\phi}c_b + b_t\right)^{1-\rho_c}}{1-\rho_c}, \tag{3.32}$$

where $c_b \geq 0$ is the threshold consumption level that measures the degree to which bequests are considered as luxury goods (Ameriks et al., 2011; Ding et al., 2014). Without considering any uncertainty, c_b is the level of consumption that makes the member indifferent to 1 dollar additional consumption or 1 dollar bequeath, i.e. if the member's consumption is below this level, they do not want to leave any bequests. Smaller values of c_b means members would want to leave bequest at a lower rate of consumption. $c_b = 0$ means preferences over consumption and bequests are homothetic and members are equally risk-averse over consumption and bequests. This gives the same expression as the bequest function in Cocco et al. (2005).

$\phi \in [0, 1)$ captures the strength of the member's bequest motive when bequest has kicked in (i.e. the wealth level is high enough to allow the member to consume at least c_b). The larger the value of ϕ, the stronger the member's bequest motive.

Family state and leisure:
Hubener, et al. (2013) consider the joint utility of couple, and the trade-off between leisure and consumption in their model. This gives interesting insight into how people place value on lifestyle in addition to spending. Consider the mix of employment types as people choose to work part time or full time. They forgo some income streams for more time spent with family. From a trustee's perspective, the focus would be more on the trade-off between consumption and bequest as it is very hard to quantify the value of leisure for our members and take that into account when we design the members' default fund. Please refer to appendix 5.1 for the technical details of how Hubener, et al. (2013) consider leisure in a multi-period utility function.

3.2.4 Non-time-separable utility function

Habit persistence:
Constantinides (1990) explores an interesting investors' behavioural property called habit persistence by allowing for adjacent complementarity in consumption. This essentially drives a wedge between the relative risk aversion (RRA) of the investors' and their EIS. There are historical evidence such as Hansen and Jagannathan (1991) and Ferson and Constantinides (1991) supporting the concept of habit persistence. The key formulation of a utility function with habit persistence is to measure consumption c_t relative to a time dependent subsistence level of consumption x_t. Constantinides (1990) defines x_t as an exponentially weighted sum of past consumption. This implies that investors place higher utility on gradually changing consumptions over time compared to sudden increase/decrease or more fluctuating consumptions patterns. Practically, it suggests that people can change their consumption habits gradually and find large changes difficult to deal with.

Recursive Utility function:
There is a more complex multi-period utility function with Epstein-Zin preferences called recursive utility function. Unlike the time additive utility function discussed in section 3.2.2, the recursive utility function is non-time-separable. Epstein and Zin (1989) is an extension of the expected utility theory that separates the two consumption preferences: risk aversion ρ_c and the inter-temporal substitutability EIS. This is then more powerful in explaining investors' behaviour as investors' EIS can be unrelated to their level of risk aversion. Please refer to appendix 5.2 for the technical details of Epstein-Zin preferences and the recursive utility function.

3.2.5 Relative or absolute measure

Utility outcomes of consumption can be measured in either absolute or relative terms compared to a consumption floor. Both Kingston and Thorp (2005) and Ganegoda and Bateman (2008) recognise the importance of a consumption floor for retirees.

Some possible options for consumption floors for retirees can be the age pension payments, or lifestyle benchmarks such as the ASFA modest and comfortable living standards.

There are generally two ways to express the relative consumption level:

- Convert the consumption figures into multiples of the consumption floor
- Calculate the difference between the consumption figures and the consumption floor

Kingston and Thorp (2005) and Ganegoda and Bateman (2008) specify a simple HARA utility function in the form below:

$$u(c_t) = \frac{(c_t - c^*)^{1-\rho_c} - 1}{1 - \rho_c} \tag{3.33}$$

where c^* is the consumption floor and $c_t > c^*$. When consumption falls closer to c^*, utility goes to zero and marginal utility approaches infinity. This reflects retirees' motivation to keep their consumption above the predetermined standard of living throughout retirement.

We recognise that sustainable consumption floors are different for individuals with different wealth levels. It is difficult for trustees to define an appropriate consumption floor standard to apply to the whole fund when a fund consists of many members with different balances.

3.3 Aggregation problem

It is straight forward to maximise the utility of the outcome for an individual member. However, when we are dealing with thousands of members and trying to design a default fund that suits the portfolio of our memberships, the aggregation of individual's utility outcomes becomes a problem. At the micro level, conditions like symmetry and separability may hold, while at the macro level they may not. This is known as the aggregation problem. There are a lot of considerations around ethical issues for members' equity and fairness.

An important question to ask is how should we weight individual outcomes in within the total utility of the fund? Should we aggregate each member's expected utility? This means members with higher balance, thus higher income streams, would be weighted higher in the aggregate utility function. Should we equally weight each member's outcome by normalising the expected utility score using the size of their balances?

The answer could be different for different funds depending on their membership profiles. As a result, it might be more appropriate for each super fund to decide which approach to adopt.

Deaton and Muellbauer (1980) discuss about PIGLOG model which was developed to treat aggregate consumer behaviour as if it were the outcome of decisions by a rational representative consumer.

4 Utility Function Design

In this section, we will discuss a number of considerations for the design of the MDUF v1 with our proposed solutions.

It would be intuitive to attempt to discover the preferences of each individual member of a fund, just as it would be intuitive to discover more about their financial situation. This is effectively the process of financial advice. However regulatory (limitations around the ability to tailor defaults) and structural issues (industry-wide shortage of advisers, as well as policy / industry focus on maintaining costs) mean that it is unlikely that funds will manage default members on this basis in the near future. The MDUF v1 project attempts to detail and mathematically represent a sensible set of preferences to assume on behalf of default members.

4.1 Practical considerations of the MDUF v1

1. Is it appropriate to maximise expected utility for optimal design of a default fund?
   Yes.

2. Should we focus on terminal wealth (lump sum) or lifetime consumption (income)?
   Lifetime income is considered a more appropriate focus for super fund trustee's acting on behalf of, but often with limited insight into, their members. Additionally this aligns with the focus of government and policymakers.

3. Should we include bequest motive in addition to consumptions?
   Yes. It is reasonable to assume members place value on bequest especially reversionary benefits to spouse.

4. What members information do we take into account?
   The basic information collected by super fund: age, gender, account balance, contributions.

5. Do we need different functions for subgroup of members?
   Not for the MDUF v1 as this will complicate the matter.

6. Consider liquidity constraint?
   Not for the MDUF v1 as this will complicate the matter.

7. Consider drawdown control (i.e. design rules which limit the size of the possible cumulative change in account value)?
   Not for the MDUF v1 as this will complicate the matter. The simple analysis below shows that a big drawdown on members' superannuation investment typically has smaller impact on their retirement incomes.

Table 4.1: Impact of drawdown on retirement income

Instant drawdown (DD)	Lumpsum $= 250,000$	Lumpsum $= 500,000$	Lumpsum $= 1,000,000$
Age = 67			
5%	2%	3%	4%
10%	5%	6%	7%
20%	9%	12%	15%
30%	14%	19%	23%
Age = 77			
5%	3%	4%	4%
10%	6%	8%	8%
20%	11%	15%	17%
30%	18%	22%	25%
Age = 87			
5%	4%	4%	5%
10%	7%	8%	9%
20%	14%	16%	18%
30%	21%	25%	27%

It is obvious that in retirement, a big drawdown on members' superannuation investment always has smaller impact on their retirement incomes through spreading over multiple periods. The impact of a drawdown on members with lower account balances (in this case the lump sum at retirement) is lower than the ones with higher balances when we taking into account age pension as a buffer. The later the big drawdown occurs at someone's retirement, the stronger will be the impact on their retirement income.

4.2 Parameterisation

1. Candidates for the time preference discount factor (β)

   - Forecast on price inflation based on CPI data

   - Forecast on wage inflation based on AWOTE data

2. Degree of risk aversion (ρ)

Paper	Utility Function	Parameter	Supporting evidence

Tobin and Dolde (1971)	CRRA defined over life-cycle consumption with borrowing constraint (liquidity constraint)	$\rho_c = 1.5$	Chose 1.5 to fit the observed life cycle savings patterns
Mehra and Prescott (1985)	CRRA defined over life-cycle consumption	$\rho_c < 10$	Concluded based on numerous studies in the past showing ρ_c is always estimated to be smaller than 10. This relies on the estimated 'price of risk' computed by Friend and Blume (1975) which is $\rho > 1$.
Gourinchas and Parker (2002)	CRRA defined over life-cycle consumption	$\rho_c < 2$	Calibrated by the life-cycle model using Consumer Expenditure Survey data
Chetty (2006)	CRRA defined over life-cycle consumption and labour supply	$\rho_c < 2$	Calibrated by the life-cycle model using data on labour supply behaviour
Schechter (2007)	CRRA defined over life-cycle consumption. Distinct between risk aversion for consumption and wealth. Meyer and Meyer (2005) show that ρ_c for consumption is about 5 times higher than the one for wealth ρ_w	$\rho_c = 1.9$	Calculated using survey data on income and experimental data on bet choice in a risk game for rural Paraguayan households and assuming individuals can not save. Can not save means income and consumption are identical.
		$\rho_c > 1,000$	Same method but assuming individuals can save. This means This gives empirical evidence for narrow bracketing
Yogo (2009)	CRRA defined over life-cycle consumption	$\rho_c = 5$	Calibrated by the life-cycle model based on the Health and Retirement Study
Ameriks et al. (2011)	CRRA defined over life-cycle consumption and end-of-life utility from bequsets	$\rho = 2, 3, 5, 10$	Selected based on other literatures: $\rho_c < 2$ (Gourinchas and Parker, 2002), $\rho > 3$ for a lot of asset pricing studies.
Lockwood (2014)	CRRA defined over life-cycle consumption and bequests	$\rho_c = 2.5$	Calibrated by life-cycle model to match retiree's savings including bequest motives and long-term care insurance choice. Model results range from 2.00 to 2.62.
Janecek (2002)	CRRA defined based on personal investment wealth	$\rho_w = 30$	For average investor based on empirical evidence from a number of experimental results.

$\rho_w = 20$	For investors with enough experience based on empirical evidence.
$\rho_w > 300$	For investors with little risk-taking experience and distaste for risk endeavors based on empirical evidence.

Table 4.2: Summary of Risk aversion parameter considered in academic literature

- **Age:** Morin and Suarez (1983) argues that older investors are more risk averse than younger investors. Bajtelsmit and Bernasek (2001) and Wang and Hanna (1997) have some debate about this finding. This could be very important to consider since we will be looking at pre-retirement and post-retirement investors.

- **Consumption and bequest:** Lockwood (2014) suggested to separate the consideration of risk aversion over consumption and bequest. It is more realistic to assume bequests are luxury goods and members are less risk-averse over bequests than over consumption. This should be acknowledged in the model.

- **Loss and gain:** Kahneman and Tversky (1979) suggested 'risk seeking' for losses and 'risk aversion' for gains based on their studies on behavioural economics. In our context, a 'loss' means running out of money during retirement and a 'gain' means being able to leave a bequest. They introduced the concept of loss aversion which hypothesizes that individuals place more weight on losses than can be explained by risk aversion. Their experiments found about a 2 for 1 weight for losses versus gains.

- **Constant or changing:** Blanchett et al. (2014) find significant evidence that risk aversion is time-varying. The way risk aversion of the investors change is related to their changing expectations of the future market rather than the experienced market returns. This would have important implications for investors' preference on asset allocations between growth and income assets under different situations.

3. EIS ($\frac{1}{\alpha}$)

    - Depends on the risk aversion parameter if time-separable utility function is used.

    - Needs to be estimated separately if non-time-separable utility function is used.

4. Strength of bequest motive (k)

    - It is difficult to determine appropriate values for k with current knowledge as empirical studies are inconclusive on how retirees view bequest (Brown, 2001; Lockwood, 2014). One way to determine k is to perform sensitivity analysis with respect to k.

5 Appendix

5.1 Utility of family state and leisure

Hubener, et al. (2013) introduce a time budget Θ that separates working hours π_t^i, commuting times $\pi_{t,trav}^i$, child care $\theta_{s,t}^i$ and leisure time l_t^i.

$$\Theta = \pi_t^i + \pi_{t,trav}^i + \theta_{s,t}^i + l_t^i \tag{5.1}$$

where $i = x, y$ denote man and woman in the family; s is the variable for family status that considers never married, married couple, divorced, widowed and the number of children.

The utility funciton at time t then becomes:

$$U(c_t, c_{t+1}, \dots, c_T) = \sum_{\tau=t}^{T} \beta^{\tau-t} \Big(\prod_{j=t}^{\tau-1} p_j \Big) \Big\{ p_\tau \frac{(\frac{c_\tau}{\varphi_s} l_\tau^\theta)^{1-\rho_c}}{1-\rho_c} + k(1-p_\tau)\frac{B_\tau^{1-\rho_c}}{1-\rho_c} \Big\} \tag{5.2}$$

$$V_t = \mathbb{E}_t[U(c_t, c_{t+1}, \dots, c_T)] = \mathbb{E}_t \Big[\sum_{\tau=t}^{T} \beta^{\tau-t} \Big(\prod_{j=t}^{\tau-1} p_j \Big) \Big\{ p_\tau \frac{(\frac{c_\tau}{\varphi_s} l_\tau^\theta)^{1-\rho_c}}{1-\rho_c} + k(1-p_\tau)\frac{B_\tau^{1-\rho_c}}{1-\rho_c} \Big\} \Big] \tag{5.3}$$

$$\max_{c_t, c_{t+1}, \dots, c_T} V_t = \max_{c_t, c_{t+1}, \dots, c_T} = \mathbb{E}_t \Big[\sum_{\tau=t}^{T} \beta^{\tau-t} \Big(\prod_{j=t}^{\tau-1} p_j \Big) \Big\{ p_\tau \frac{(\frac{c_\tau}{\varphi_s} l_\tau^\theta)^{1-\rho_c}}{1-\rho_c} + k(1-p_\tau)\frac{B_\tau^{1-\rho_c}}{1-\rho_c} \Big\} \Big] \tag{5.4}$$

where $l_\tau = \sqrt{l_\tau^x \times l_\tau^y}$ is the effective leisure considering both spouses. φ_s is a scalling factor used for normalized total consumption based on the number of adults and children present in the household. θ measures the preference for leisure with $\theta < 1$ represents declining marginal utility for leisure time.

5.2 Recursive Utility function

Epstein-Zin preference (Epstein and Zin, 1989) is an extension of the expected utility theory that separates the two consumption preferences: risk aversion ρ_c and the inter-temporal substitutability EIS.

We first look at the standard expected utility time-separable preference defined as:

$$V_t = \mathbb{E}_t \Big[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau) \Big] \tag{5.5}$$

This can be defined recursively as:

$$V_t = u(c_t) + \beta \mathbb{E}_t[V_{t+1}] \tag{5.6}$$

Or equivalently with a scaling of $(1-\beta)$ for consumption at time t:

$$V_t = (1-\beta)u(c_t) + \beta \mathbb{E}_t[V_{t+1}] \tag{5.7}$$

Epstein-Zin preference involves defining recursively $F(\cdot)$ as an 'aggregator' function that maps over current (known) consumption c_t and a certainty equivalent $R_t(V_{t+1})$ of tomorrow's utility V_{t+1}.

$$V_t = F(c_t, R_t(V_{t+1})) \tag{5.8}$$

where

$$R_t(V_{t+1}) = G^{-1}(\mathbb{E}_t[G(V_{t+1})]) \tag{5.9}$$

with $F(\cdot)$ and $G(\cdot)$ increasing and concave. $\mathbb{E}_t[\cdot]$ is the conditional expectation given all the information we know at time t. Most literature considers simple functional forms for $F(\cdot)$ and $G(\cdot)$ as follows:

$$\alpha > 0 : F(c, z) = [(1-\beta)c^{1-\alpha} + \beta z^{1-\alpha}]^{\frac{1}{1-\alpha}} \tag{5.10}$$

$$\rho_c > 0 : G(x) = \frac{x^{1-\rho_c}}{1-\rho_c} \tag{5.11}$$

The inverse of $G(x)$ becomes:

$$\rho_c > 0 : G^{-1}(y) = [(1-\rho_c)y]^{\frac{1}{1-\rho_c}} \tag{5.12}$$

$R_t(V_{t+1})$ can then be expressed in the following form:

$$\rho_c > 0 : R_t(V_{t+1}) = \left[(1-\rho_c)\mathbb{E}_t\left[\frac{V_{t+1}^{1-\rho_c}}{1-\rho_c}\right]\right]^{\frac{1}{1-\rho_c}} = \mathbb{E}_t\left[V_{t+1}^{1-\rho_c}\right]^{\frac{1}{1-\rho_c}} \tag{5.13}$$

$$\rho_c = 0 : R_t(V_{t+1}) = \exp(\mathbb{E}_t[\log(V_{t+1})]) \tag{5.14}$$

$$V_t = F(c_t, R_t(V_{t+1})) = \left[(1-\beta)c_t^{1-\alpha} + \beta\mathbb{E}_t\left[V_{t+1}^{1-\rho_c}\right]^{\frac{1-\alpha}{1-\rho_c}}\right]^{\frac{1}{1-\alpha}} \tag{5.15}$$

$$V_T = F(c_T, 0) = c_T \tag{5.16}$$

In general, ρ_c is the relative risk aversion coefficient and α is the inverse of the elasticity of inter-temporal substitution (EIS) for deterministic variations. A special case is when $\rho_c = \alpha$ or if consumption is deterministic, we will then have the usual standard time-separable expected discount utility with $EIS = \frac{1}{\alpha}$. If $\rho_c > \alpha$, early resolution of uncertainty is preferred when the expected utilities of two lotteries are the same. A higher α gives a lower EIS and thus it means the person becomes less willing to substitute consumption inter-temporally. This means they will be less sensitive in changing their consumption plan when they expect changes in future real interest rates/real investment returns. T is the maximum possible time for the lifetime horizon and V_T is then the utility at time T and $R_t(V_{t+1})$ does not exist.

Bequest motive: If we add in bequest motive, it will lead to the following expression. This is discussed in Horneff et al. (2008), Horneff et al. (2009) and Blake et al. (2014).

$$V_t = F(c_t, R_t(V_{t+1})) = \left[(1-\beta)c_t^{1-\alpha} + \beta\mathbb{E}_t\left[p_t V_{t+1}^{1-\rho_c} + (1-p_t)\left(\frac{k}{1-\rho_c}\right)\left(\frac{B_{t+1}}{k}\right)^{1-\rho_c}\right]^{\frac{1-\alpha}{1-\rho_c}}\right]^{\frac{1}{1-\alpha}} \tag{5.17}$$

$$V_T = F(c_T, 0) = \left[c_T^{1-\alpha} + \beta\mathbb{E}_T\left[\left(\frac{k}{1-\rho_c}\right)\left(\frac{B_{T+1}}{k}\right)^{1-\rho_c}\right]^{\frac{1-\alpha}{1-\rho_c}}\right]^{\frac{1}{1-\alpha}} \tag{5.18}$$

PART II: Member's Default Utility Function v1 (MDUF v1)

1 Introduction

This part shows an illustration of our proposed Member's Default Utility Function v1 (MDUF v1) that can be tailored for specific use in the superannuation industry. The developed MDUF v1 uses a life-cycle model that takes into account investment and mortality risks.

Guiding principles of the MDUF v1 were:

1. Utility function is an appropriate tool to reflect our members' preference.
2. MDUF v1 should be a sensible representation of the retirement income problem, viewed through the lens of a trustee.
3. The focus is on lifetime consumption (income) rather than terminal wealth.
4. End-of-life residual benefit value is considered to recognise the value placed by members.
5. Rational behaviour of our members are assumed. Behavioural biases are not part of the consideration.
6. Where possible we stay close to the mainstream academic literature.
7. The framework chooses a simpler approach where possible to increase the ability for the industry to make use of MDUF v1.
8. The framework is tractable so it can be used for fund design.

Specific considerations regarding members's preferences, incorporated into the design of MDUF v1 include:

1. Higher income through life
2. Smooth income over time
3. Outliving savings is bad outcome
4. Residual benefit has value
5. People are risk averse

MDUF v1 does not capture liquidity preferences. There is currently no dominant stream of research on how to incorporate liquidity preferences into a preference function. This would be a valuable consideration in a version 2 project. In the meantime we would advise that super funds consider incorporating formal liquidity limits (modelled through life) into their retirement solution design.

It would be desirable for the MDUF v1 to perform a "straw man" role for super funds trying to determine appropriate objectives for their default members. In this respect the MDUF v1 would represent a proposal which a super fund could argue against in justifying their own set of objectives. MDUF v1 can be used in many ways including the following:

1. To compare a number of solutions by ranking them based on their utility scores.
2. To seek optimal solution within a single product. For example, by recommending an optimal consumption path and asset allocation inside an account-based pension.
3. To seek optimal solution across multiple products. For example, by recommending an

optimal mix of products.

4. To quantify the value of advice by calculating the utility gain of better retirement strategies.

5. To quantify the utility cost of suboptimal solutions.

6. To help super funds with project prioritisation.

Note that notation in Part II is self-consistent and there can be inconsistency in notation between Part I and Part II. The structure of this paper is as follows. Section 2 describes the model framework and introduces notation. Section 3 briefly outlines the solution technique that is used to maximise lifetime utility. Case studies with numerical results are shown in Section 4. A series of sensitivity analyses are performed and included in the Appendix. The last section proposes recommendations with respect to the life-cycle model to be used in practice.

2 Framework

The utility function is designed in a way to jointly solve for optimal lifetime consumption patterns and allocation to risky assets. This can be represented by O_t which denotes the vector that consists of all choice variables at time t including drawdown and asset allocation, i.e.

$$O_t = (c_t,\ \omega_t) \quad 0 < t \leq T, \tag{2.1}$$

where c_t is the annual consumption level at time t, and ω_t is the weight of the wealth invested in risky asset at time t.

The life-cycle utility maximisation problem is:

$$\max_{\{O_t\}_{0\leq t\leq T}} \mathbb{E}_0 \left[\sum_{t=0}^{T} \beta^t \left\{ {}_tp_x\ u(c_t) + {}_{t-1|}q_x\ v(b_t) \right\} \right], \tag{2.2}$$

subject to

$$c_t \geq 0, \tag{2.3}$$
$$\omega_t \in [0,\ 1], \tag{2.4}$$
$$b_t \geq 0, \tag{2.5}$$

where x is the inception age of a particular cohort, β is the subjective utility discount factor that captures the retiree's time preference, and $\mathbb{E}_0$ is the expectation operator with respect to time 0 (equivalently, age x).

The value of the subjective utility discount factor (i.e. β) has received a number of debates in the academic literature. Some argue that a rational individual should place equal value throughout life, so they believe that the subjective utility discount factor should be equal to 1 (see e.g., Broome, 1991; Elster, 1986; Rawls, 2009; Becker and Murphy, 1988). Looking from an individual's point of view, some philosophers (such as Zemach, 1987; Parfit, 1993) describe an individual as "a succession of overlapping selves related to varying degrees of memories" so they believe it is rational to discount future utility (Frederick, 1999). The values of subjective utility discount factor used in the literature are largely variable (Frederick, 1999). Through the lens of a trustee, it is appropriate to assume $\beta = 1$. This means we focus on a sustainable retirement income strategy through life rather than catering to potential myopic (short-sighted)

biases which could place retirement outcomes at risk. In addition, from the view of trustees that represent many members of different cohorts, the intergenerational equity is an important issue, i.e. having a less than 1 utility discount factor would mean less value given to those who survive to older ages.

$u(c_t)$ is the utility function defined over consumption at time t:

$$u(c_t) = \frac{c_t^{1-\rho}}{1-\rho}. \tag{2.6}$$

$v(b_t)$ is the utility function defined over end-of-life residual benefit (or bequest)[3]:

$$v(b_t) = \frac{b_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho}, \tag{2.7}$$

where b_t is the level of wealth at time t which equals the amount of residual benefit if the person dies between $t-1$ and t. $\rho > 0$ is the level of risk aversion. Higher ρ means more risk aversion of the investors. ϕ is the strength of residual benefit motive. Higher ϕ means stronger residual benefit motive. ${}_tp_x$ is the probability of being alive at age $x+t$ conditional on being alive at age x. ${}_{t-1|}q_x$ is the probability of dying between age $x+t-1$ and $x+t$ conditional on being alive at age x.

Therefore, the life-cycle maximisation problem in our framework is

$$\max_{\{O_t\}_{0\le t\le T}} \mathbb{E}_0\left[\sum_{t=0}^{T}\left\{{}_tp_x\ u(c_t) + {}_{t-1|}q_x\ v(b_t)\right\}\right], \tag{2.8}$$

where notation is the same as in the previous paragraphs.

The form of the lifetime utility then becomes:

$$U_0 = \mathbb{E}_0\left[\sum_{t=0}^{T}\left\{{}_tp_x\ \frac{c_t^{1-\rho}}{1-\rho} + {}_{t-1|}q_x\ \frac{b_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho}\right\}\right], \tag{2.9}$$

The dynamics of wealth is

$$b_{t+1} = (b_t + P_t - c_t)\left(1 + \tilde{R}_{t+1}\right), \tag{2.10}$$

where P_t is the amount of Age Pension entitlement received at time t if considered, and $\tilde{R}_t$ is the stochastic return p.a. of the investment portfolio from time $t-1$ to t, which is assumed to follow a Normal distribution.

2.1 More Explanation on Utility Function and Residual Benefit Motive

We provide more intuitive explanation on the utility function and residual benefit motive function we use for the MDUF v1.

2.1.1 Utility Function

The utility we adopt is a CRRA utility function of consumption (see Section 3 in Part I for a detailed review of different classes of utility functions). The general pattern of the adopted utility function with respect to consumption is shown in Figure 2.1.

[3] In the remaining part, we use the residual benefit instead of bequest to soften the wording.

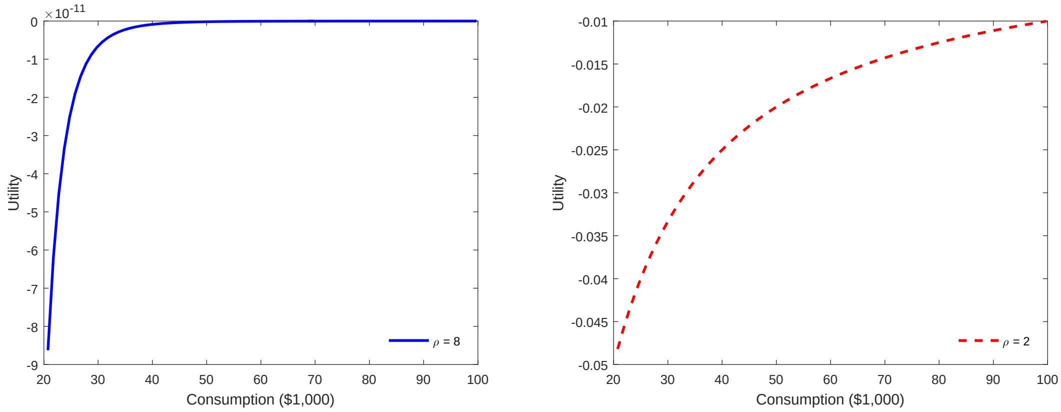


Figure 2.1: Utility function with respect to consumption.

We have the following observations for the utility function:

- The higher the consumption level, the higher the utility.
- Utility is a ranking of preferences - the absolute level does not mean anything.
- ρ is called the risk aversion parameter that represents people's conservativeness. For risk-averse people (i.e. $\rho > 0$), the penalty of \$1,000 less consumption is higher than the reward of \$1,000 dollar more consumption. Table 2.1 shows an example on how utility changes when consumption is \$1,000 lower or higher. We see that when the risk aversion parameter is 8, the utility is reduced by 19% when the consumption is lowered from \$40,000 to \$39,000, whereas the percentage increase in utility is less than 16% when the consumption is increased from \$40,000 to \$41,000. At a less risk-averse level (e.g. $\rho = 2$), the penalty put on consumption reduction is only slightly larger than the gains from consumption increase.

Table 2.1: Percentage changes in utility when consumption changes from \$40,000.

	\$1,000 Less	**\$1,000 More**
$\rho = 8$	-19.39%	15.87%
$\rho = 2$	-2.56%	2.44%

- The higher the risk aversion parameter, people are more afraid of lower consumption. From Table 2.1 we can see that a higher risk aversion parameter causes a more pronounced utility loss resulting from consumption reduction than utility gain from consumption increase of the same amount.

2.1.2 Residual Benefit Motive Function

The motive on leaving residual wealth at death to his/her beneficiary is also a CRRA function. The residual benefit motive shares the same risk aversion parameter (i.e. ρ) with the utility function of consumption. In the following we explain key observations of the residual benefit motive function.

- The higher the wealth level at death (as long as positive), the higher the utility from leaving values in the residual benefit.

- Ceteris paribus, a higher residual benefit motive parameter represents preference to leaving more in the residual benefit.

- For example, in a one-period model where $\phi = 0.5$ the individual obtains equal utility from \$1 consumption and from leaving \$1 in the residual benefit; in a one-period model where $\phi = 0.8$ the individual will only consume 20% of wealth and leave 80% in the residual benefit.

3 Optimal Strategy Solution Technique

This section focuses on the complex computational techniques required in solving for optimal dynamic strategies. To fully understand the detail in this section, it would require knowledge in business school postgraduate courses and equivalent or higher levels. Readers who find the detailed solution technique too difficult can skip this section and proceed to the next section on case studies.

Using the Bellman equation, the utility maximisation problem in Equation (2.8) can be expressed in the following recursive equation:

$$V(b_t) = \max_{O_t,\ a} \left\{ u(c_t) + \mathbb{E}_t \Big[p_{x+t}\ V(b_{t+1}) + q_{x+t}\ v(b_{t+1}) \Big] \right\}, \tag{3.1}$$

where $V(b_t)$ denotes the maximised utility based on information up to time t, p_{x+t} is the annual survival probability of an individual aged $x + t$, and q_{x+t} is the annual death probability of an individual aged $x + t$. This recursive equation in essence ensures that the optimum of the optimum is the global optimum.

First-order conditions and envelop conditions are then derived based on the Bellman Equation (3.1). See the remaining part of this section for details on first-order condition and envelop condition. Optimal solution is directly coded based on the derived first-order condition. The envelop condition is useful in that it gives us the formula for an important component in the first-order condition.

The solution technique consists of two separate stages. The first stage is backward induction. In this stage, we start from the terminal period, define grid points of wealth dynamics, and calculate optimal choice using first-order conditions and envelop conditions. The second stage is forward simulation. Once the optimal solution for each grid point of wealth is obtained in the first stage, forward simulations are then used to demonstrate optimal paths of consumption and asset allocation. The resulting dynamics of wealth can then be calculated using these forward simulations and the corresponding optimal values in the choice variables.

4 Case Studies

Our case studies are centred around the account-based pension (ABP) for non-homeowner single males. The reason for focusing on non-homeowners is that taking into account housing assets would require a good analysis of the housing asset dynamics and would complicate our illustrations. Our case studies are for singles instead of couples, as incorporating joint mortality and spouse information would further complicate the model.

Two financial assets and two retirement products are included in our case studies. The two financial assets are a risk-free asset and a risky asset (equity). We assume that the real risk-free

rate is fixed at r_f p.a. for the horizon considered. Real returns for risky assets are assumed to follow an identical and independent normal distribution, i.e.:

$$R_t \sim N(\mu_R,\ \sigma_R^2).$$

The two additional retirement products include the Age Pension and life annuities. The Age Pension entitlements are means-tested. The rules are detailed in Section 4.2. The features and pricing methods of life annuities are described in Section 4.3.

We show numerical results for the above proposed life-cycle model with and without taking into account the Age Pension and life annuities.

Mortality rates are sourced from the Australian Life Tables 2010-12 by Australian Government Actuary[4]. Survival curves and death distributions for males and females are shown in Figure 4.1.

For illustration purposes, we only show case study results for males in this paper. The results for females can be easily replicated by using the corresponding female mortality rates.

4.1 Case 1: Base Case

We start with a scenario where the Age Pension is not taken into account and life annuities are not available. Parameter values are shown in Table 4.1.

Table 4.1: Parameter values for base-case analysis. Sources are cited in brackets.

Parameter	Explanation	Value	Source
r_f	Risk-free rate	0.00%	Assumption
μ_R	Mean equity return	5.00%	Assumption
σ_R	Standard Deviation of equity return	15.00%	Assumption
ρ	Risk aversion	8	MDUF v1 Specification
ϕ	Residual benefit motive strength	0.83	Lockwood (2014)
b_0	Initial wealth ($1,000)	500	Assumption
P_t	Age Pension Entitlement ($1,000)	0	Assumption

The residual benefit motive strength parameter ϕ is chosen to be 0.83. This means the propensity to consume, which is calculated as the ratio of annual consumption to residual benefit is equal to $1 - \phi$ (17%). A 17% propensity to consume means if a retiree has 100 dollars, he/she will tend to spend 17% and save 83% in their accounts.

This choice of parameter is based on Lockwood (2014). We incorporate this parameter into our framework and justify the appropriateness of using this parameter in MDUF v1 through a simple model illustration. The detail of the illustration can be found in Appendix 9.2. The illustration shows that the ratio of total consumption to residual benefit value changes with respect to different residual benefit motive strength and the length of planning horizon. The key message is that it allows MDUF v1 to place a lower value of the residual benefit than the long-term income stream that it could generate. This prevents the development of bequest prioritisation strategies - “live on a low level of income and maximise the residual benefit”. This

[4]The Australian Life Tables 2010-12 can be downloaded via `http://www.aga.gov.au/publications/life_table_2010-12/`

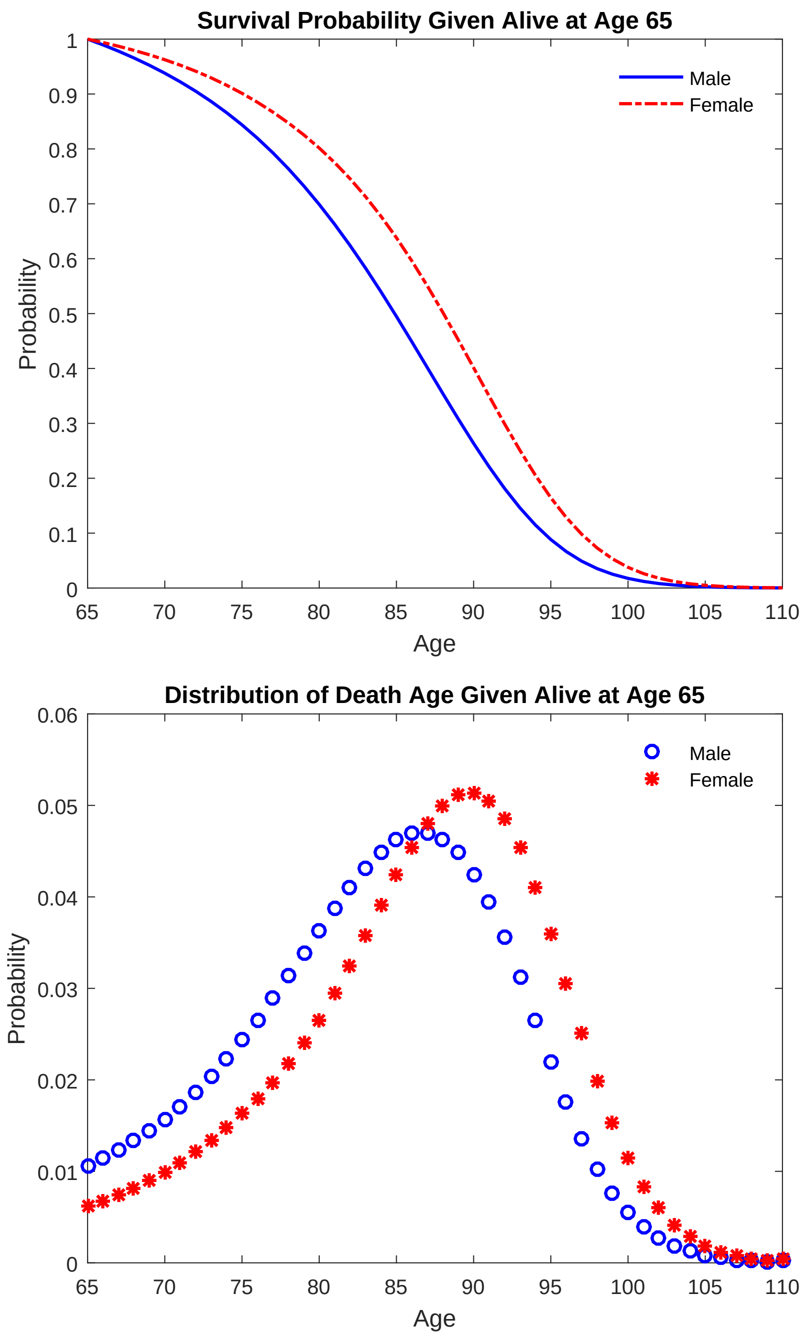


Figure 4.1: Survival curves and death distributions conditional on surviving to age 65.

is an important consideration of MDUF v1. As a result, we believe that the choice of $\phi = 0.83$ is appropriate to reflect our members' preference around residual benefit.

The risk aversion parameter ρ is chosen to be 8. This choice of parameter is a combined result of literature reviews and model calibration. We have reviewed a range of relevant academic literature in the field of CRRA utility defined over life-cycle consumption and bequest. A summary of the risk aversion parameters ρ used in a wide range of literature can be found in Table 4.2 in Appendix 9.3. Our finding is that the choice of ρ in most studies fall within the range of 1 to 10.

To further consider the appropriate parameter for MDUF v1, we developed and calibrated a lifecycle model using the assumptions in Table 4.1 to show the resulting year-to-year changes in

retirement income based on $\rho = 2, 5, 8, 10$.

Some members of the MDUF Working Group expressed a view that the potential for year-to-year changes in retirement income of more than 10% would not be palatable to super fund members. There was broad agreement on this amongst the MDUF Working Group. Figure 4.2 shows the mean and 95% confidence intervals of the year-to-year percentage changes in consumption obtained in our MDUF model for different risk aversion parameter values. We observe that a risk aversion parameter of 8 delivers fairly reasonable variability in year-to-year consumption changes consistent with the views of the MDUF Working Group. This level is also within the range (1 to 10) used in many academic studies (for instance, Ameriks et al., 2011; Yogo, 2009; Mehra and Prescott, 1985; Friend and Blume, 1975).

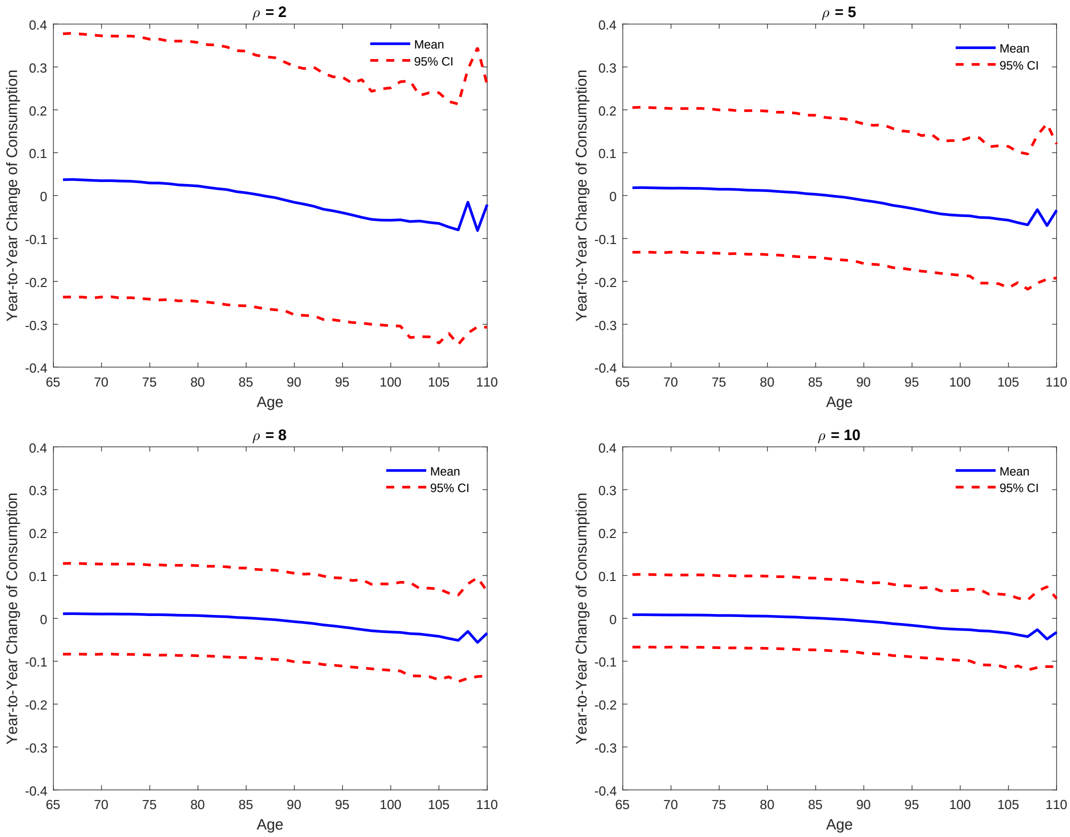


Figure 4.2: Consumption year-to-year percentage changes for different risk aversion parameters.

The optimal average consumption paths, asset allocation, wealth paths, and propensity to consume (consumption as a proportion of wealth) are shown in Figure 4.3.

We can see from Figure 4.3 that the optimal allocation to risky asset is a constant proportion of wealth, which is 33.95%. This constant proportion is due to the CRRA specification of utility function. The optimal average consumption path slightly increases for the first 20 years. This is because retirees tend to sacrifice their consumption in the first few years for potentially more accumulation in their wealth, which is also motivated by their desire to hedge the risk of living longer than expected. The average consumption path starts to decrease from age 90 onward, which is largely due to the consideration of (idiosyncratic) longevity risk, or survival probability scaling. The average wealth path shows a generally decreasing pattern, which is a mixed result of risky asset growth and wealth draw-down. The red dashed lines indicate the 95% confidence intervals. Variations are caused by investment risk and idiosyncratic mortality risk.

Another observation is the very stable (as shown by the almost deterministically increasing

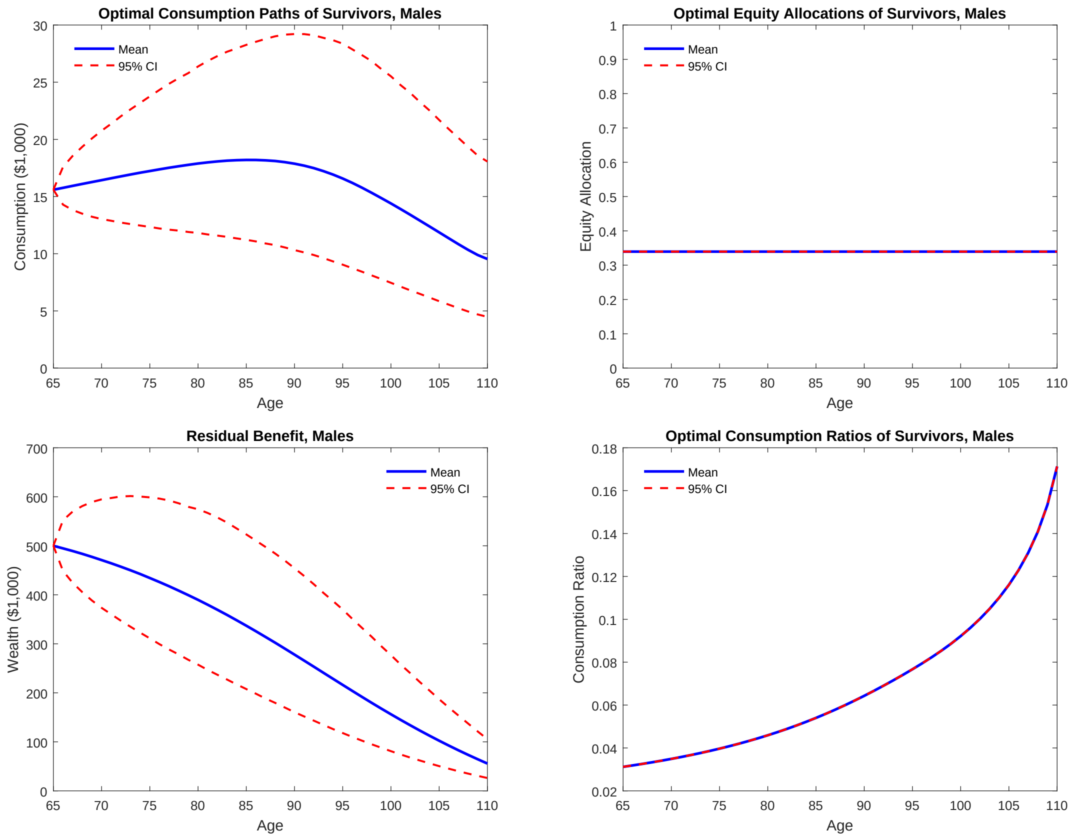


Figure 4.3: Optimal average consumption, asset allocation and wealth paths.

pattern) propensity to consume, which is calculated as the ratio of consumption to wealth. The consumption ratio starts from 3%-4% (similar to the traditional rule of thumb Bengen (2004)) at retirement and more than doubles when the retiree reaches age 88. At the maximum attainable age (i.e. 110 in our study), the propensity to consume is 17%, which is equal to 1 minus the residual benefit motive strength (ϕ). This relationship is not a coincidence, but indeed is due to the definition of residual benefit motive strength. The increasing pattern is very comparable to the ABP minimum drawdown rules, under which the consumption ratio is 4% at age 65 and increases to 14% at age range 95-110.

4.2 Case 2: The Impact of the Age Pension

In this section, we incorporate the Age Pension and follow the Australian Government's eligibility rules. The amount of Age Pension entitlement is means-tested, i.e. depending on the current asset value and future expected incomes[5]. We formulate the following tests to determine the Age Pension entitlement:

- Asset test

$$P_t^A = \max\left(\bar{P} - \tau^A \max\left(b_t - \underline{b},\ 0\right),\ 0\right), \tag{4.1}$$

  where P_t^A is the Age Pension entitlement under the asset test, $\underline{b}$ is the asset test threshold for full pension, $\bar{P}$ is the full Age Pension payment rate, and τ^A is the taper rate for asset test.

[5]See the Australia Government's Age Pension website at `https://www.humanservices.gov.au/customer/services/centrelink/age-pension`

- Income test under deeming rule

$$P_t^I = \max\left(\bar{P} - \tau^I \max\left(r_1 \min(b_t,\ \underline{b}_1) + r_2 \max(b_t - \underline{b}_1,\ 0) - \underline{I},\ 0\right),\ 0\right), \tag{4.2}$$

where P_t^I is the Age Pension entitlement under the income test, $\underline{b}_1$ is the deeming threshold, below which a lower deeming rate r_1 is applied and above which a higher deeming rate r_2 is applied. $\underline{I}$ is the income test cut off point. τ^I is the taper rate for income test.

- Combining the two tests

$$P_t = \min\left(P_t^A,\ P_t^I\right). \tag{4.3}$$

- Values of these parameters are listed in Table 4.2.

Table 4.2: The Age Pension eligibility and payment rates as at 1 July 2016. Single and non-home owner rates are used. Information is sourced from the Australian Government Department of Human Service website. Note that Effective from 1 January 2017, there are a few changes to these parameter values. The results in this technical paper do not incorporate these updates.

Parameter	**Explanation**	**Value**
$\bar{P}$	The Full Age Pension payment rate (p.a.)	\$22,721.4
τ^A	Taper rate under the asset test	0.0015
$\underline{b}$	Threshold for full pension under asset test	\$360,500
$\underline{b}_1$	Threshold for different deeming rates under income test	\$49,200
r_1	Lower deeming rate	1.75%
r_2	Higher deeming rate	3.25%
$\underline{I}$	Income test cut off point (p.a.)	\$4,264
τ^I	Taper rate under the income test	0.5

The optimal average consumption paths, asset allocation, wealth paths, and propensity to consume (consumption as a proportion of wealth) are shown in Figure 4.4. Compared with Case 1, we see a generally higher proportion of wealth that should be invested in equity, especially in early years. The optimal allocation to equity is 80% and shows an almost linearly decreasing trend as the retiree ages. This decreasing pattern with respect to age is different from the base case results, but as the individual survives to an old age the allocation to equity converges to the constant proportion in Case 1 (i.e. 33.95%). This is because in the terminal period there is no future Age Pension, resulting in the same problem as in Case 1. Therefore the optimal asset allocations in the two cases are the same for the terminal period. The greater the distance from the terminal period, the higher the Age Pension entitlement is expected to be, resulting in larger differences in the asset allocations in the two cases.

The dollar amount of consumption is higher than the general consumption level in the base case. The bell shape of average consumption is also a feature of this case study. See more comparative analysis of this in Section 4.4. As expected, the consumption ratio is significantly higher than the base case where there is no Age Pension. We also see some variation around consumption ratios when the Age Pension is taken into account. The reason is that the optimal consumption not only depends on the current wealth level but also on another source - the Age Pension.

The wealth level is indeed generally higher than that in Case 1, due to the Age Pension supplements. The average wealth level almost decreases to around \$150k in the terminal period. For a comparison, the average wealth in Case 1 decreases to around \$50k in the terminal period.

The amount of the Age Pension received is shown in Figure 4.5. The figure shows the average and 95% confidence interval of the Age Pension entitlements. The average amount of the Age

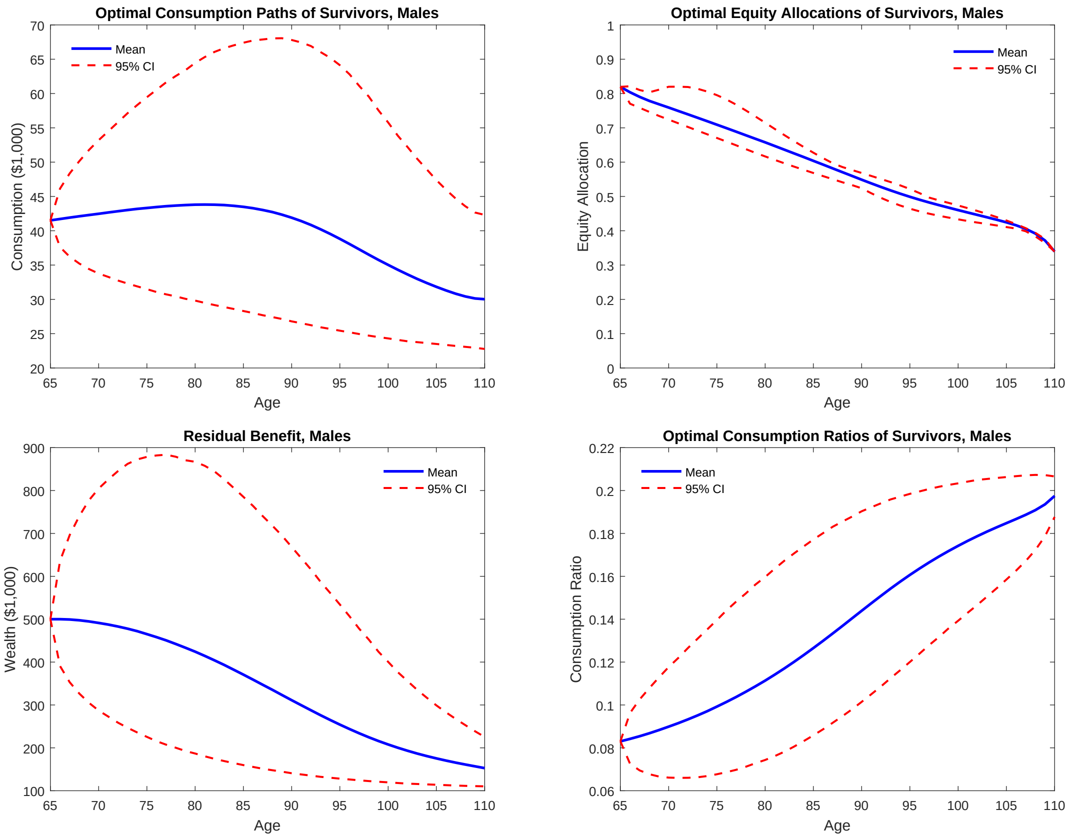


Figure 4.4: Optimal average consumption, asset allocation and wealth paths when the Age Pension is taken into account.

Pension received slightly decreases for the first five years and starts to steadily increase onward. The slight decreasing pattern for the first five years is largely due to the non-linear relationship between the Age Pension entitlement and wealth level. After the first five years, the average value of the Age Pension entitlement starts to increase because the level of wealth is likely reduced due to draw-down so the amount of the Age Pension entitlements becomes high.

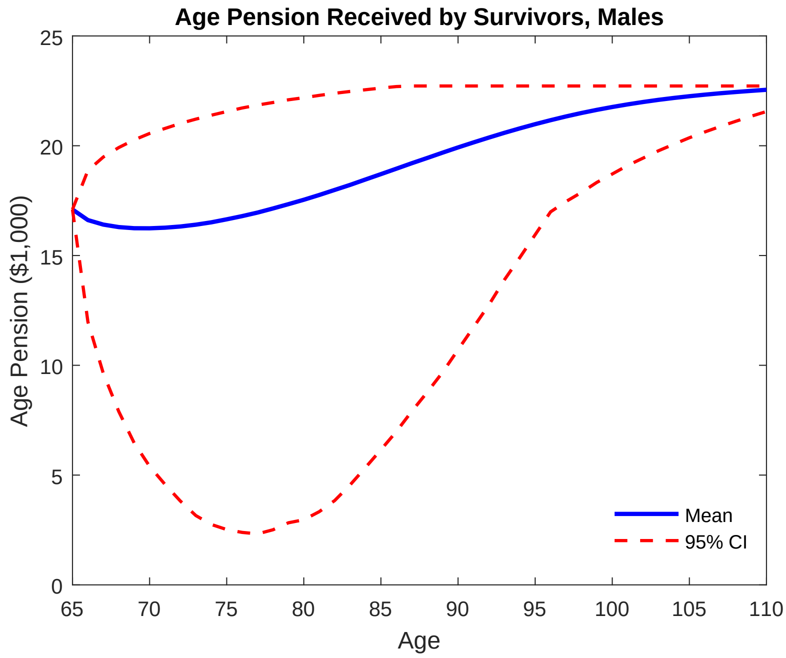


Figure 4.5: The Age Pension Entitlements in Case 2.

To summarise, the Age Pension functions as a safety net for retirees. It increases the general level of consumption and motivates retirees to invest more wealth in risky assets. The welfare improvement from having access to the Age Pension is analysed in Section 5.

4.3 Case 3: The Role of Life Annuities

In this case, we investigate the role of life annuities in retirees' retirement planning, in particular focusing on consumption smoothing and longevity risk protection.

We use life annuities that provide inflation-linked annual benefits. For illustration purposes, we assume life annuities are priced using the risk-free interest rate. Other forms of loadings, such as expense loading, profit loading, and the cost of capital requirements, are not explicitly included in the pricing.

Note that this case study aims to provide an illustration of the impact of life annuities on retiree's lifecycle strategy, so the numerical results should not be used in any way for specific financial advice.

The annuitisation ratio, calculated as the ratio of annuitised wealth to the initial wealth, is one of the control variables as stated in Section 2. We find the annuitisation ratio along with consumption and asset allocations that would generate the highest expected lifetime utility, as formulated in Section 2.

Based on the Age Pension eligibility rules for lifetime income streams[6], we formulate the following tests to determine the Age Pension entitlement:

- Asset test

$$P_t^A = \max\left(\bar{P} - \tau^A \max\left(b_t^p + b_t^a - \underline{b},\ 0\right),\ 0\right), \tag{4.4}$$

  where

$$b_t^a = \max\left(b_0^a - \frac{b_0^a}{\mathring{e}_x} \times t,\ 0\right), \tag{4.5}$$

  b_t^p and b_t^a are the time t value of assets in account-based pension and annuity respectively. b_0^a is the purchase price of annuity at time 0. $\mathring{e}_x$ is the life expectancy of the member age x when the annuity is purchased at time 0.

- Income test

$$P_t^I = \max\left(\bar{P} - \tau^I \max\left(r_1 \min\left(b_t^p,\ \underline{b}_1\right) + r_2 \max\left(b_t^p - \underline{b}_1,\ 0\right) + \max\left(I_t^a - \frac{b_0^a}{\mathring{e}_x},\ 0\right) - \underline{I},\ 0\right),\ 0\right), \tag{4.6}$$

  where I_t^a is the annuity income at time t and $\frac{b_0^a}{\mathring{e}_x}$ is the deduction amount under the Age Pension rule for annuity.

- Combining the two tests

$$P_t = \min\left(P_t^A,\ P_t^I\right). \tag{4.7}$$

Figure 4.6 shows the level of Certainty Equivalent Consumption (CEC) for different annuitisation ratios. The relationship shows a bell shape. As shown in the figure, the optimal proportion of wealth that should be annuitised is around 45% for retirees with half a million dollars in their super fund at retirement. Annuitising more than 80% of the initial wealth would result in a utility level even worse than without any annuities. Key reasons include the residual benefit motive, the crowd-out effect of the Age Pension, and the assumed low interest rate in annuity pricing. Annuitising too much wealth will result in a very low value in the residual benefit, which places a large penalty on the lifetime utility if the residual benefit motive is relatively high. The Age Pension also provides longevity protection, so it can have a substituting impact on the demand for life annuities. In our case a low interest rate is assumed in the pricing, which means the price of life annuity is expensive, so the demand for life annuities is also reduced.

The optimal average consumption paths, asset allocation, wealth paths, and propensity to consume (consumption as a proportion of non-annuitised wealth) are shown in Figure 4.7. We see that the optimal expected consumption path becomes flatter than that in Case 1 and Case 2. This reflects the consumption smoothing function of purchasing life annuities. The average annual consumption at very old ages is above $40,000 p.a. and the lower 95% confidence interval of annual consumption at very old ages is around $33,000. By comparison, the average consumption at very old ages in Case 2 is above $30,000 and the lower 95% confidence interval is around $25,000. This shows the longevity risk protection provided by life annuities.

We can see that the expected consumption ratio as a proportion of non-annuitised wealth for early retirement in Case 3 almost doubles that in Case 2. The key reason is higher consumption level as well as lower dollar amount of non-annuitised wealth, which can be confirmed by the residual benefit in the bottom left panel of Figure 4.7.

[6]See the Australia Government's Age Pension website at https://www.humanservices.gov.au/customer/enablers/income-streams

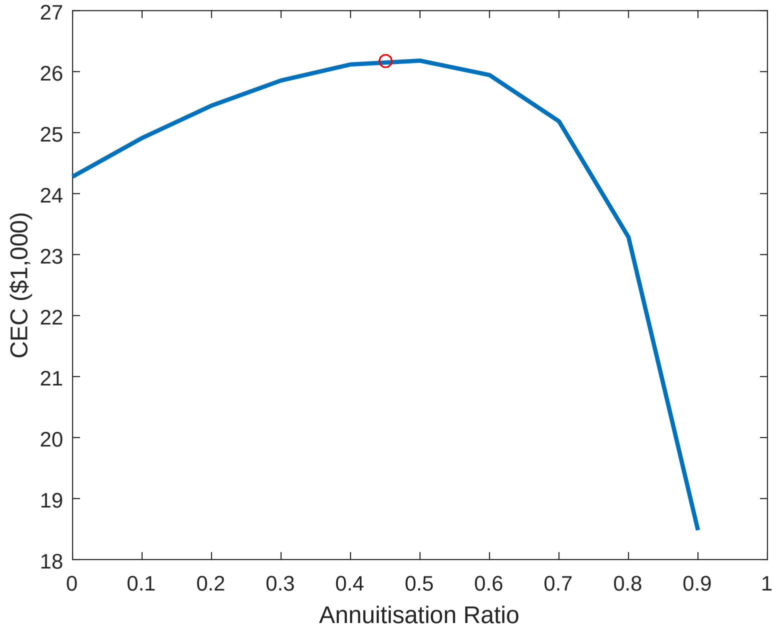


Figure 4.6: Certainty Equivalent Consumption (CEC) for different annuitisation ratios. The highest CEC is marked in red circle.

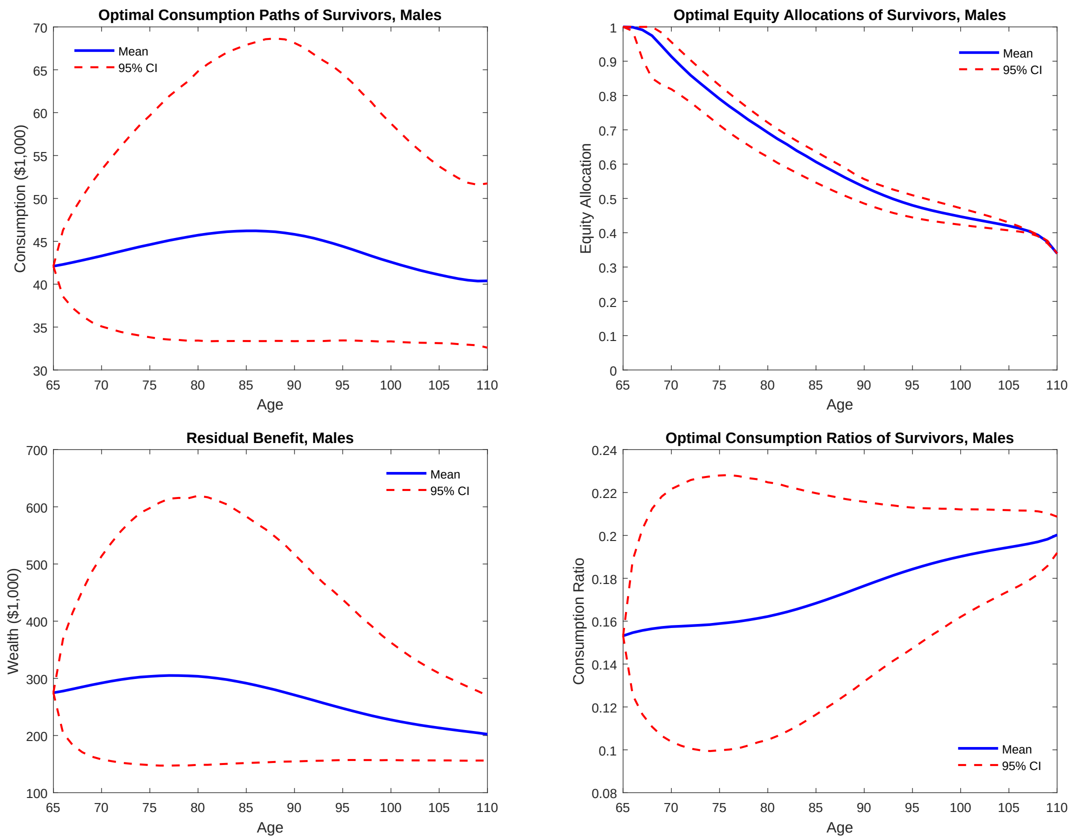


Figure 4.7: Optimal consumption, asset allocation and wealth paths when the Age Pension and life annuities are taken into account. Note that equity allocation and consumption ratio are quoted as the proportion of non-annuitised wealth.

The equity allocation as a proportion of non-annuitised assets in Case 3 is generally higher than that in Case 2. It starts at 100% and decreases all the way down to 33.95%. Comparisons of asset allocations for the three cases are discussed in Section 4.4.3.

The Age Pension entitlements at different ages are shown in Figure 4.8. The figure shows the average and 95% confidence intervals of the Age Pension entitlements. The average amount of the Age Pension received shows a similar increasing pattern in general as in Case 2, except that in Case 2 the average of the Age Pension entitlement decreases for the first few years. A detailed comparison of the Age Pension entitlements between the two cases is shown in Section 4.4.4.

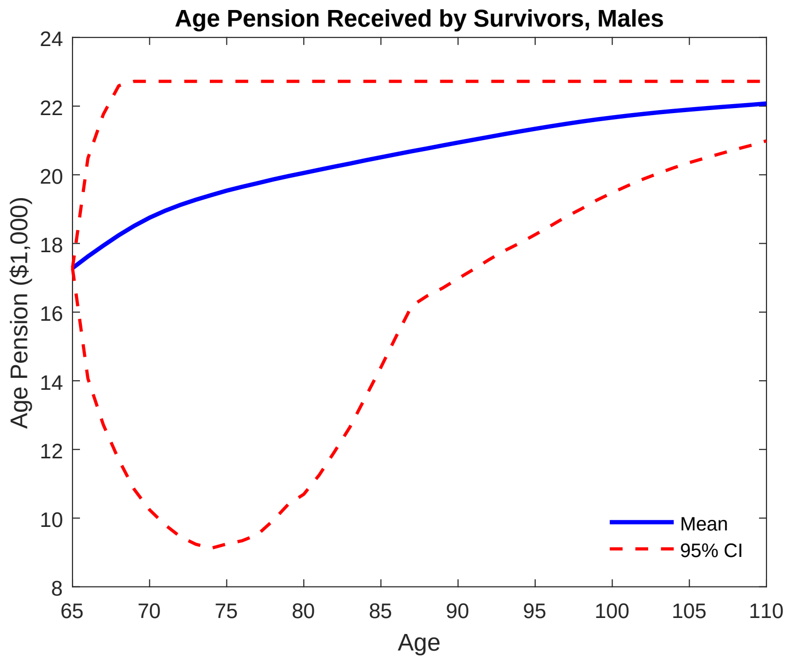


Figure 4.8: The Age Pension Entitlements in Case 3 (with access to life annuities).

4.4 Comparative Analysis

This section provides a comparative analysis across the three cases.

4.4.1 Consumption Comparison

The Association of Superannuation Funds of Australia (ASFA) provides retirement standards at the level of living a modest lifestyle and of living a comfortable lifestyle, respectively. As of 2016 for 65-year-old homeowner singles, the modest lifestyle standard is $23,797 p.a. and the comfortable lifestyle standard is $43,184.[7] We compare our expected consumption levels in the above three cases with ASFA retirement standards. Note that the ASFA standards are for homeowners whereas our case studies focus on non-homeowners. We therefore encourage caution in directly comparing the results in our case studies with ASFA standards. The results are shown in Figure 4.9.

[7] Data are obtained from ASFA's website via `http://www.superannuation.asn.au/resources/retirement-standard`, on 24 May 2016.

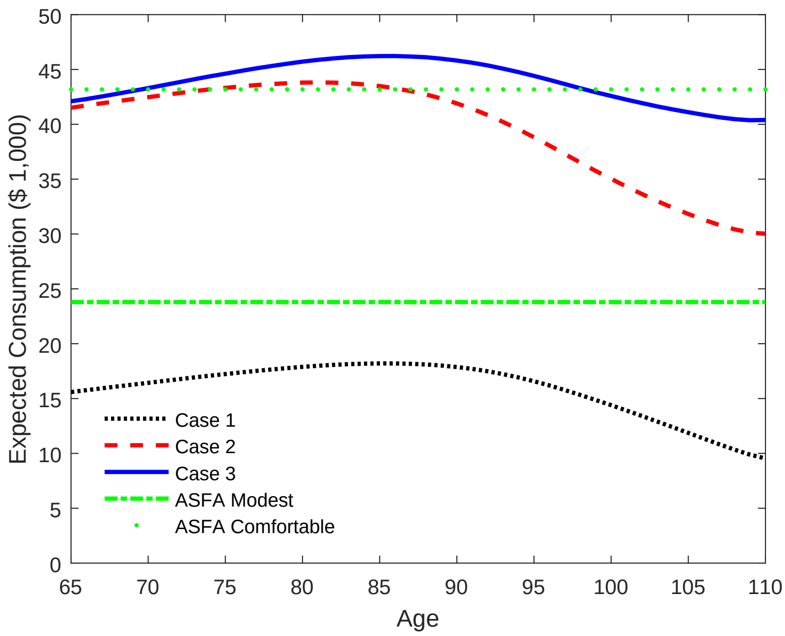


Figure 4.9: Comparison of expected consumption levels for different cases and ASFA retirement standards. In Case 1, retirees have no access to the Age Pension or to life annuities; In Case 2, retirees have access to the Age Pension; In Case 3, retirees have access to the Age Pension as well as life annuities.

As indicated by the black dotted line in Figure 4.9, for a 65-year-old male with half a million dollars retirement savings, the expected consumption level is below the ASFA modest standard for most of the time in Case 1, i.e. if the retiree has no access to the Age Pension or life annuities. When the Age Pension is accessible (Case 2), the expected consumption level is substantially higher than the ASFA Modest level and reaches the level around the ASFA comfortable standard in the first 20 years and starts to decrease for older ages, as indicated by the red dashed line. This shows that the Age Pension can provide the retiree with streamlined retirement incomes to improve the living standard. Annuitising a proportion of retirement assets can result in a higher level of consumption, especially for old ages. Due to the longevity protection provided by life annuities, the consumption levels for older ages in Case 3 (blue solid line) are higher than those in Case 2 (red dashed line).

4.4.2 Residual Benefit Comparison

Figure 4.10 compares the expected residual benefit at death for the three cases. Comparing Case 1 and Case 2, we observe higher residual benefit when the Age Pension is accessible, which stems from the positive residual benefit motive. When purchasing life annuities, the residual benefit is lower for earlier ages, but becomes higher for those who survive to very old ages, i.e. after age 97. Note that the probability of a 65-year-old Australian male surviving to age 97 is less than 5%. Taking into account survival probabilities, we can observe that life annuities increase the lifetime consumption most likely at the cost of a reduction in the residual bnefit.

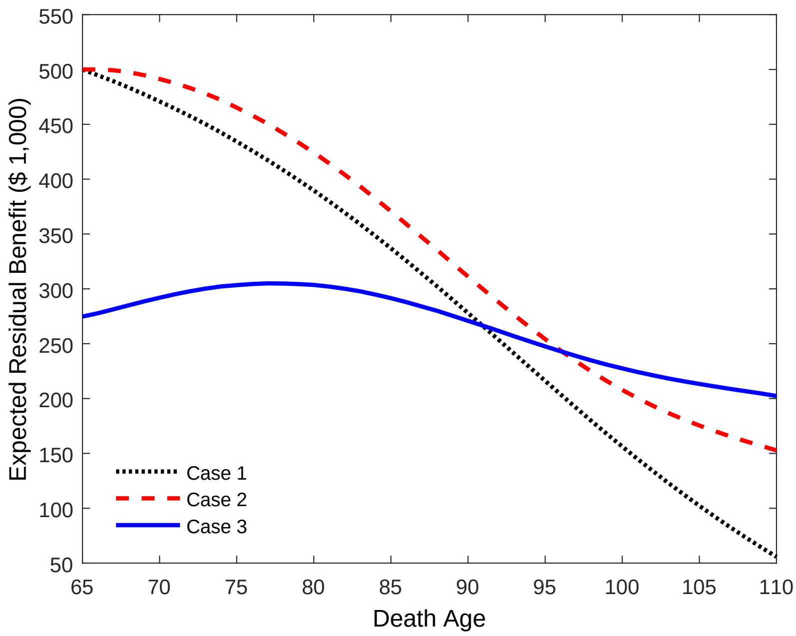


Figure 4.10: Comparison of expected residual benefit at death. In Case 1, retirees have no access to the Age Pension or to life annuities; In Case 2, retirees have access to the Age Pension; In Case 3, retirees have access to the Age Pension as well as life annuities.

4.4.3 Asset Allocation Comparison

Asset allocations are compared across the three cases. The results are shown in Figure 4.11. We observe substantially higher allocations to risky assets if the Age Pension is taken into account. This is largely due to the income protection provided by the Age Pension. When life annuities are accessible the optimal allocation to equities as a proportion of non-annuitised wealth is higher than that in Case 2 for earlier ages, but this discrepancy becomes very small after Age 80. Due to the fact that the optimal strategy is to annuitise around 45% wealth at retirement, the optimal dollar amount of equity investment in Case 3 is indeed lower than that in Case 2. The bottom panel of Figure 4.11 shows the dollar amount of allocations to different assets at retirement. We observe lower allocations to risky asset in Case 3 than in Case 2.

4.4.4 The Age Pension Entitlement Comparison

The amount of the Age Pension entitlement is compared between Case 2 and Case 3. The results are shown in Figure 4.12. We can see the advantages of life annuities in terms of the Age Pension entitlements. The advantages are twofold: the average amount of the Age Pension is higher for earlier ages (e.g. before age 98 in our case studies); downside variations are lower except for very old ages (i.e. after age 105). The enhancement in the Age Pension entitlement for annuities stems from each of the two tests: under the income test, part of annuity benefit is not considered as income but as the return of capital; under the asset test, annuity asset is assumed to depreciate linearly until reaching the life expectancy.

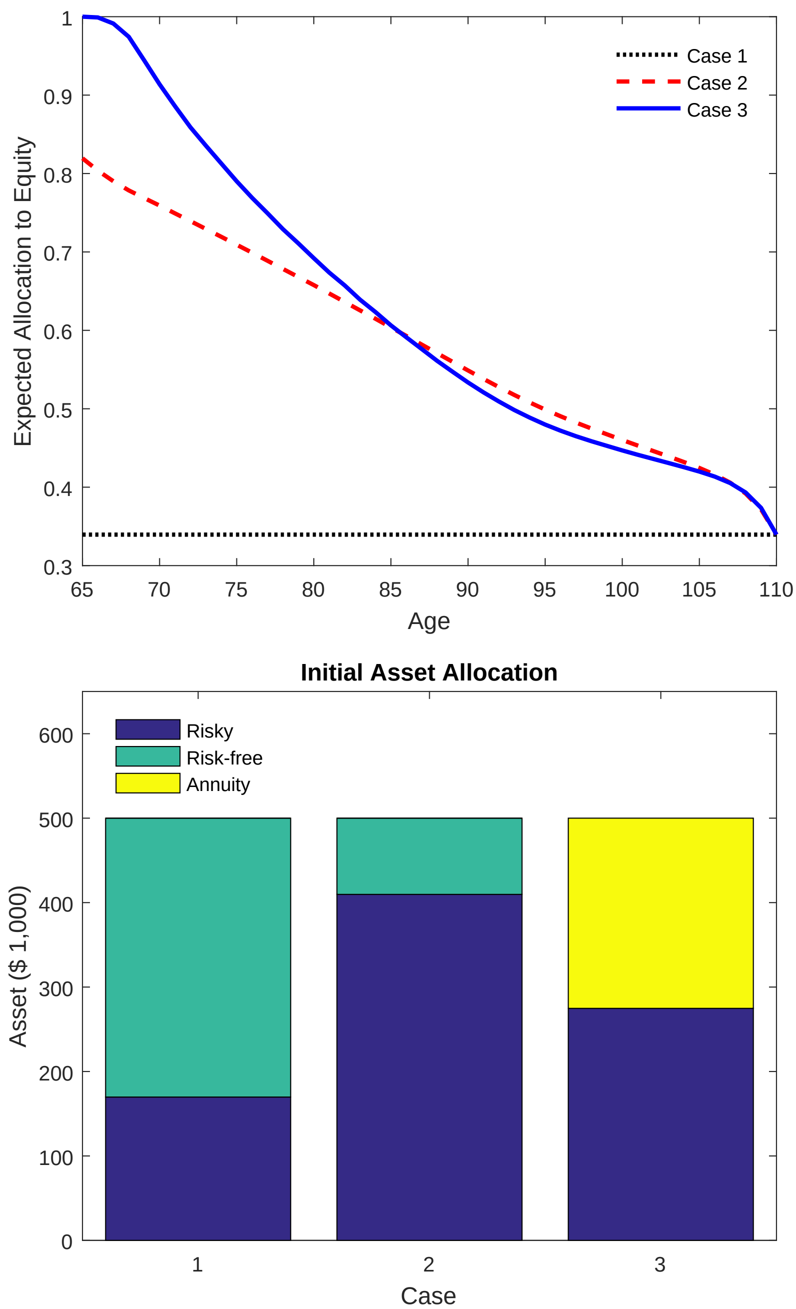


Figure 4.11: Comparison of expected allocations to equity for different cases. The top panel shows the expected allocation to risky asset as a proportion of non-annuitised wealth. The bottom panel shows the dollar amount allocation to risky asset, risk-free asset and life annuity at retirement. In Case 1, retirees have no access to the Age Pension or to life annuities; In Case 2, retirees have access to the Age Pension; In Case 3, retirees have access to the Age Pension as well as life annuities.

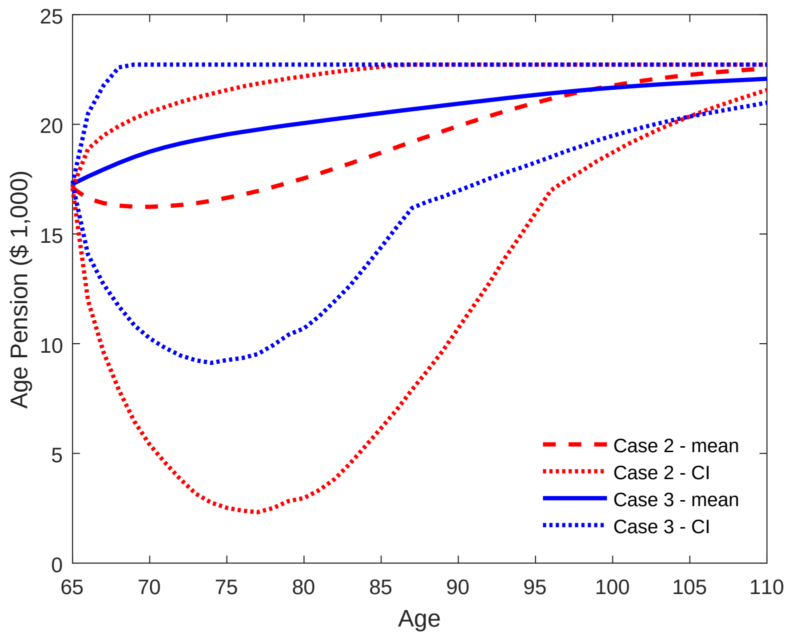


Figure 4.12: Comparison of the Age Pension entitlements for different cases. In Case 2, retirees have access to the Age Pension only. In Case 3, retirees have access to the Age Pension as well as life annuities.

5 Welfare Analysis

Utility can be used as a scoreboard of an individual's lifetime welfare, which is a ranking instead of an absolute score. It is difficult to perform quantitative comparisons directly based on utility scores.

There are indeed other measures related to utility which are widely used to investigate welfare gains or losses of different retirement strategies. These measures can also be used in our case to assess retirees' welfare gains of having access to the Age Pension provided by the Australian government and to life annuities. This section describes these key measures and shows numerical results based on our model set-up.

5.1 Metrics on Welfare Gains

5.1.1 Certainty Equivalent Consumption (CEC)

Certainty Equivalent Consumption is calculated as the consumption level in the one-period utility function, i.e. equating $\frac{c_t^{1-\rho}}{1-\rho}$ to the lifetime utility calculated for the strategy using Equation 2.9. CEC, in essence, is a monotonic transformation of the lifetime utility. A higher level of the lifetime utility also corresponds to a higher CEC level. Note that CEC does not necessarily convey information of the actual level of consumption.

5.1.2 Wealth Gap (WG)

For a pair of two cases, the wealth gap is calculated as the additional amount in the initial wealth in one case that can result in the same level of optimal life-time utility in the other case. When we compare Case 2 vs. Case 1 and Case 3 vs. Case 2, the wealth gap measures the dollar amount of welfare gains, reflected in the initial wealth level, of having access to the Age Pension and life annuities, respectively.

5.1.3 Extra Annual Return (EAR)

For a pair of two cases, the extra annual return is calculated as the additional annual return of the fund investment performance in one case that can result in the higher level of optimal life-time utility in the other case. When we compare Case 2 vs. Case 1 and Case 3 vs. Case 2, the extra annual return measures welfare gains, reflected in terms of annual investment gains, of having access to the Age Pension and life annuities, respectively.

5.2 Results

Tables 5.1 and 5.2 show the optimal lifetime utility and the three metrics used for assessing welfare gains of having access to the Age Pension and life annuities. Note that the results are for retirees with initial wealth of $500k. The results would vary for different wealth levels.

Table 5.1: Value functions and CECs of the three cases.

	Case 1	**Case 2**	**Case 3**
Optimal Lifetime Utility (10^{-10})	-202.33	-0.29	-0.17
CEC ($1,000)	9.51	24.28	26.18

Table 5.2: Welfare gains of having access to the Age Pension (AP) and/or life annuities (LA). Initial wealth is $500k.

	AP vs. Nil	**AP + LA vs. AP**	**AP + LA vs. Nil**
WG ($1,000)	775.75	105.55	875.64
EAR (%)	6.36	1.13	7.04

We can see a substantial improvement in CEC level when the Age Pension is incorporated. For example, CEC in Case 2 increases by 155% compared to that in Case 1. This confirms that the Age Pension provided by the Australian government has been improving the general welfare of the retired population to a large extent.

The wealth gap of Case 2 vs. Case 1 (i.e. the Age Pension vs. Nil) is about $776,000, reflecting the substantial welfare gains of having access to the Age Pension. In other words, the Age Pension provided by the Australian government provides the same welfare gains as if providing a lump sum subsidy of $776,000 at retirement to retirees who have half a million dollars in their account balance. On top of having entitlements to the Age Pension, the provision and efficient use of life annuities generates an extra wealth gap, which is calculated to be about $106,000. We still see significant wealth improvement by providing life annuities due to the protection of longevity risk, though this amount is relatively small compared to that of the Age Pension vs. Nil. Key reasons for the relatively small welfare gains from life annuities are (1) the Age Pension

is funded via tax whereas life annuities cost a large lump sum at retirement; (2) longevity risk is largely reduced by the Age Pension, leaving smaller room for life annuities to take a role.

Comparing EAR in Case 3 and Case 1, we can conclude that the pension fund needs to grow at an extra rate of 7.04% p.a. in order to meet the welfare generated from the Age Pension and life annuities. This extra annual return is indeed very substantial in the current financial environment.

6 Key Discussions

There has been some good robust debate around a couple of specific issues with the Member's Default Utility Function ("MDUF") amongst the working group. Below is a summary of these important discussions which have helped shape the current version of MDUF v1.

To reflect on the current status of the superannuation industry, the industry is currently dominated by a focus on account balance and performance, often represented by peer group relative performance. The industry recognises that retirement income is the objective of superannuation. Secure income is valued higher than risky income. Yet guidance is lacking as to how to trade off the level of retirement income versus the certainty of that income stream.

This is where the MDUF has a defining role to play. It demonstrates that there are techniques available that help a fund assess different distributions of possible retirement outcomes. The MDUF can become an integral framework for ex-ante decision making for the fund. These two points make this project potentially have a profound impact on the superannuation industry.

It is important that the MDUF can be used in two ways:

1. as an objective function used to compare and assess different retirement income solutions; and

2. as an objective function used to design solutions which reflect sensible objectives for underlying fund members.

If the MDUF is to make a worthwhile contribution to the industry then it must be accessible. If we make the MDUF too complex then many groups will not use it and much of the value could be lost. The starting point is that there is currently no guide for fund design which combines level of income with security of the income stream. Given this backdrop it is considered to be better to select a more simple utility function which is used by 90% of the industry than a more complex, technically more complete, model which can only be understood by just a few funds.

It is recognised that the MDUF is far from perfect. The following framework is used to proceed.

1. Sticking reasonably closely to the mainstream academic literature on this topic. The aim is not striving to advance the academic literature on objective functions, rather trying to package up the most sensible and usable parts of the academic research on this topic.

2. Where possible keeping the model simple rather than complex. Recognising the "straw man" concept of the MDUF v1. Further enhancement can be considered in the future.

3. Everyone is welcome, indeed encouraged, to design their own objective function. Of course it would be beneficial for the whole industry if the reasoning was shared.

The working group discussions have been mainly on three topics that are presented in the

following subsections.

6.1 Placing value on the residual benefit

Philosophically it is hard to place value on something that people will not experience because they are deceased. Some policymakers view superannuation as purely retirement income and place little value on what is leftover, yet people purchase life insurance for various reasons. There are also structural issues. The superannuation system has account-based or account value origins. Members think of super as their account balance. It will take many years, even decades, to change this focus. Under such a structure members and hence trustees would place value on the leftover amount when they pass away.

We believe that, when looked through the lens of the trustee, it is important to assign some value to any residual benefit.

1. There is a distinct risk of dying early in retirement. Assuming one were to retire today at age 65: then for a male (female) there is a 1.1% (0.6%) chance of dying in the first year of retirement and a 15.6% (9.9%) chance of dying in the first decade of retirement. In these cases we believe that it would be inappropriate for a trustee to design a post-retirement solution which places no value on any residual benefit;

2. The superannuation system is designed around the individual, not the household, yet over 65% of people retire with a partner. For households with a significant income difference between the two partners the residual account value provides the retirement outcome for the surviving (low income) partner;

3. Empirical research suggests that people do place value on the bequest aspect associated with a residual benefit. One example is Lockwood (2014), a research paper which attempts to model retirement solution design to align with the empirical evidence around bequest motive. It shows very consistent annuitization result from the model compared to the empirical data when allowing for relatively modest bequest motive.

4. Residual benefit acts as a reserve pool for many life events related to aged care, health-care, travelling and family. This is an important point given we do not capture liquidity preference in MDUF v1.

Overall, we believe there is a very strong case for trustees to put some value on any residual benefit. With these considerations, the MDUF v1 is put forward with the following points:

- Acknowledge that, given the issues raised above, it is sensible to place some value on any residual benefit at death.
- Soften the wording where possible to focus on "residual benefit" rather than "bequests".
- Design and parameterise the MDUF such that the residual benefit is valued less than the income stream it can produce – thereby deterring "bequest gaming" (hoard savings / collect age pension / maximise bequest).
- Explain how the MDUF can be adapted to have a retirement income only focus.

In Appendix 9.2, we use two simple models to demonstrate that the residual benefit motive is not "bequest gaming" (i.e. reducing consumption and prioritising bequests). The two models show the impact of residual benefit put on retirement income relative to residual wealth.

6.2 Using discounted utility

There is no evidence of individuals undertaking complex modelling and determining an integrated investment and consumption decision which maximises their expected discounted utility (DU). Nonetheless, we believe this framework is appropriate to serve as the basis of a sensible objective for trustees to assume on behalf of members on which they have little insight.

The discounted utility framework (though note that our parameterisation presented in step 1 involves no discounting) is not without challenge in the academic literature, notably Frederick et al. (2002). Some of the key concerns raised in the paper are consumption independence and stationary instantaneous utility. Consumption independence means no consideration of the impact of consumption changing year-to-year and stationary instantaneous utility is the assumption that the utility function and parameters like risk aversion remain constant through life. These concerns are valid but do not dissuade us from continuing on down the discounted utility path for the following reasons:

- The DU approach embodies many sensible messages: the lifetime experience of retirement income is important, and, via the use of concave utility curves, a drop in income results in a larger loss of utility than the extra utility experienced from the same quantum increase in income. If returns were certain and we knew when we would die then a DU approach would recommend smoothed income.
- The DU approach inherently focuses on sustainable outcomes, an approach that we believe is consistent with the responsibility of trustees.
- There is a substantial amount of academic literature focusing on optimal consumption and investment decisions which have used DU as the representation of objectives.
- Most extensions or alternatives to the DU approach introduce components which significantly impact the tractability of the model and make it much harder to understand and be used by industry. Basically every additional variable introduced adds a dimension to the problem (the "curse of dimensionality" as it is known!).

Overall the DU framework is concluded to be the most appropriate approach for this project. It carries good high level messages, focuses on sustainable solutions, is relatively easy to understand, and is frequently used in academic research in optimal consumption and investment problems. In combination this provides a highly defensible base for trustees.

6.3 Reflecting a preference for smooth income

Research has identified that individuals seek smooth income (see e.g., Hall, 1978; Modigliani and Brumberg, 1954). It is likely that this would result in lower income levels. We contend that an approach that priorities cash flow smoothing comes at the risk of unsustainability. When looked at through the lens of a trustee we do not believe it appropriate to explicitly target income smoothing.

One approach is to explicitly incorporate a preference for habit formation in consumption. A detailed example is provided in Campbell et al. Another technique can be to introduce a reference level of income; such a technique is difficult across a heterogenous population thereby making this approach difficult in application. Further techniques are detailed in Asher (2007) and Montserrat et al. (2006). Many of these techniques introduce complexity and a potential degree of intractability when applying the objective function.

Additionally we observe that a decision to smooth income may be based on the view that markets mean revert over time. This does not necessarily require a change in the objective function; rather it is the underlying model for asset returns which needs to be altered (mean reverting returns implies time varying returns). The complexity of the underlying model is at the discretion / skills of the researcher. It would be imprudent to reflect market assumptions within the objective function.

We are motivated to consider whether the discounted utility framework based on CRRA can reflect a preference for smooth income. If investment returns were certain and we knew our date of death then our DU / CRRA framework would indeed recommend as optimal a strategy with constant income. However what happens in the real world environment when investment returns and mortality outcomes are uncertain? The optimal strategy would suggest a level of income that is sustainable by spreading the expected consumption evenly on a basis which accounts for the distribution of investment returns and mortality outcomes and maximises total expected utility. As a result a change in wealth due to market movements results in a change in sustainable income. This is an important message for trustees: the risk that is taken at a fund level at any time will impact the sustainable level of income not just this year but for all future years. The DU / CRRA framework provides a strong focus on sustainable income in retirement. If we acknowledge the dynamic nature of the problem we need to constantly reassess the optimal consumption and investment strategy. Any prioritisation of income smoothing is to the detriment of a sustainable retirement outcome strategy.

The coefficient of relative risk aversion represents the trade-off between a higher level of expected income and year-on-year variability in income. A higher coefficient of relative risk aversion implies a preference for smoother income at the cost of a lower level of expected income.

Overall we believe that the DU / CRRA framework is appropriate for MDUF v1. The core focus of DU / CRRA is sustainability of retirement income. The choice of the coefficient of relative risk aversion allows the consideration of the trade-off between high income preferences and smooth income preferences. We have specifically addressed this calibration issue in Section 4.1.

7 Stepping Outside the MDUF v1

The life-cycle model proposed in this paper is sophisticated and flexible. After determining a final set of parameter values, we can apply the MDUF v1 for better retirement fund design. The MDUF v1 is not perfect and there are some aspects that the function can not address, particularly liquidity preferences. We will need to understand these limitations when we apply MDUF v1. As a result, there are cases when a trustee may want to step away from the "straw man" provided in the form of MDUF v1.

Specific reasons for a trustee to step away from MDUF v1 include:

1. may want to allow for behavioural biases.

2. may have greater member insight.

The trustees that wish to account for these aspects can choose to step outside the MDUF v1 through a number of ways:

1. Changing parameter values. For example, the trustee can choose a different risk aversion parameter value ρ based on their better understanding of their members. The trustee can also turn off residual benefit motive by setting ϕ equal to 0 if they believe their members

do not value residual benefit at all.

2. Incorporating additional features into the function, such as a consumption floor, luxury bequest etc.

8 Recommendations and Conclusions

In the second part of this paper we have detailed a utility function that we recommend as the Member's Default Utility Function for Default Fund Design v1 (MDUF v1). We believe that the recommended MDUF v1 is a credible and powerful metric that can be used to address the complex retirement outcome problem.

9 Appendix

9.1 Sensitivity Analysis

In this section, we investigate the impact of parameter values on the optimal results, including consumption, asset allocation, annuitisation ratio, and Age Pension entitlement. As our annuity price is based on the risk-free interest rate, the impact of interest rate on the annuitised benefit is mixed. Therefore, we show the annuitised annual benefit in the sensitivity analysis with respect to risk-free interest rate. For sensitivity analyses with respect to other parameters, the annuity pricing based does not change so the annuitised benefit is proportional to the initial wealth.

9.1.1 Risk Aversion

We first investigate the impact of the risk aversion parameter (i.e. ρ). Results are shown in Figure 9.1. The optimal annuitisation ratio increases as the member becomes more risk-averse. More risk-averse members have a flat expected consumption path and can better utilise the Age Pension (mainly due to higher annuitisation ratios) but at a cost of generally lower consumption level. We observe substantially lower average levels of investment in equity (in terms of dollar amount as well as proportion of non-annuitised wealth) when the trustee assumes more risk-averse members, i.e. the risk aversion parameter is higher.

9.1.2 Risk-free Interest Rate

The impact of risk-free interest rate (i.e. r_f) is then investigated. The results are shown in Figure 9.2. The impact of risk-free interest rate on the optimal annuitisation ratio shows a U shape. As interest rate increases, the annuity becomes cheaper, which increases the demand of annuities. At the same time the same amount of asset generates higher annual benefits as interest rate increases, so having a bequest motive would reduce the demand of annuity. This can be confirmed in the top right panel which shows the increasing annual benefit from annuity as the risk-free interest rate increases.

The impact of risk-free interest rate on the Age Pension entitlements is also mixed. A slight increase in the risk-free interest rate makes earlier consumptions higher so the wealth level is reduced faster resulting in higher Age Pension entitlements in earlier years. The higher interest rate, however, increases wealth level in the long run, so the Age Pension entitlements are lower in later years. When the risk-free interest rate is very high (e.g. 6% p.a.), the converted annuity benefits are high enough to reduce the Age Pension entitlements to a low level, which can be observed in the black dot-dashed line in the middle right penal. We can observe a substantial reduction in equity investment if the risk-free interest rate is higher.

9.1.3 Initial Wealth

Sensitivity test results for the initial wealth (i.e. b_0) are shown in Figure 9.3. The optimal equity investment as a proportion of wealth is not sensitive with respect to initial wealth. The expected consumption level shifts upward as the initial wealth is higher. For retirees with low wealth, the optimal annuitisation ratio is much lower, which is largely due to liquidity constraints. We see that the very wealthy members (i.e. with 2 million dollars initial wealth) can annuitise 60% of their asset and better utilise the Age Pension rules for life annuities.

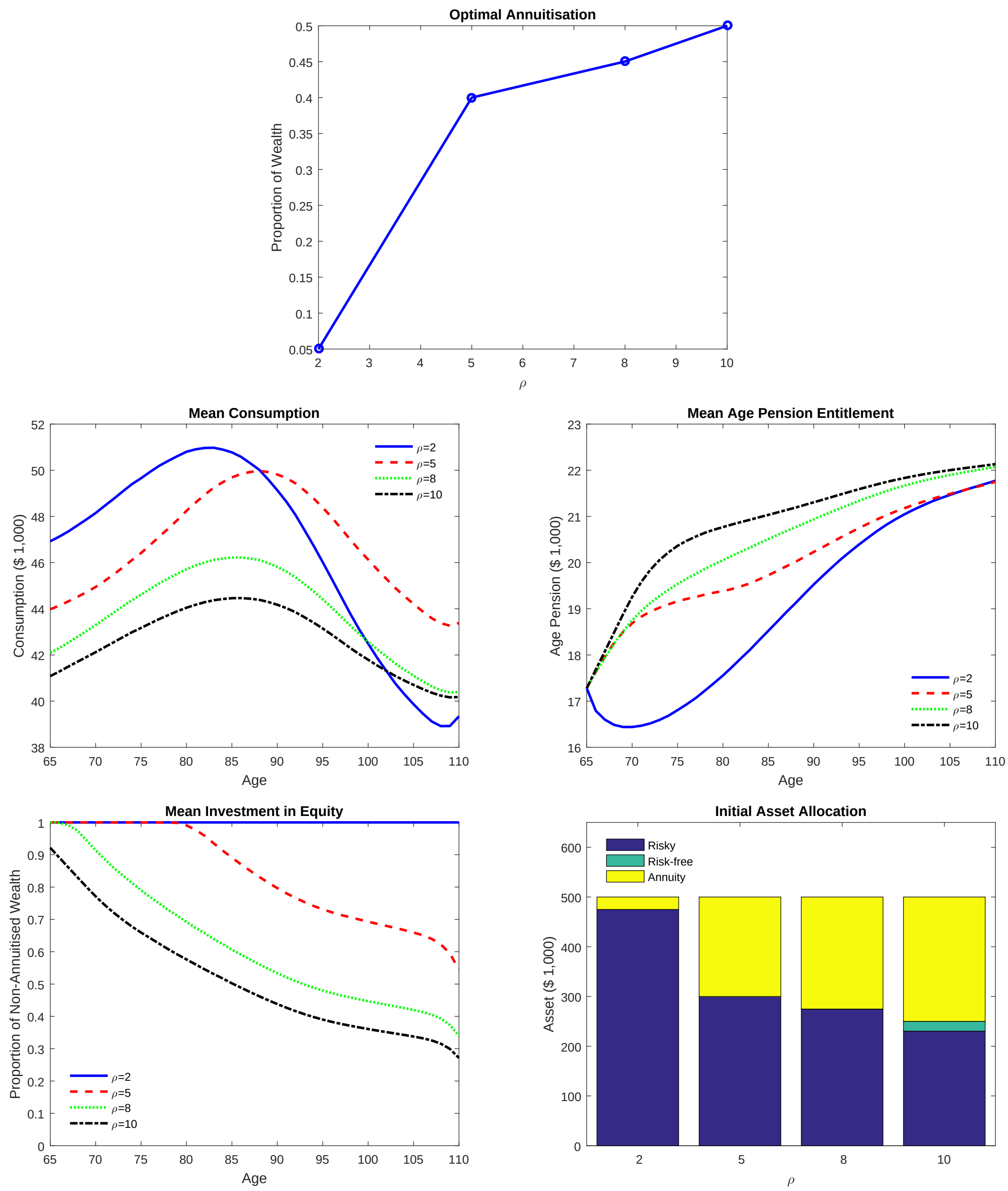


Figure 9.1: Sensitivity analysis of risk aversion parameter on the optimal annuitisation ratio (top panel), the optimal mean consumption path (middle left panel), the mean Age Pension entitlements received (middle right panel), the optimal mean proportion of non-annuitised wealth invested in equity (bottom left panel), and the asset allocation at retirement (bottom right panel).

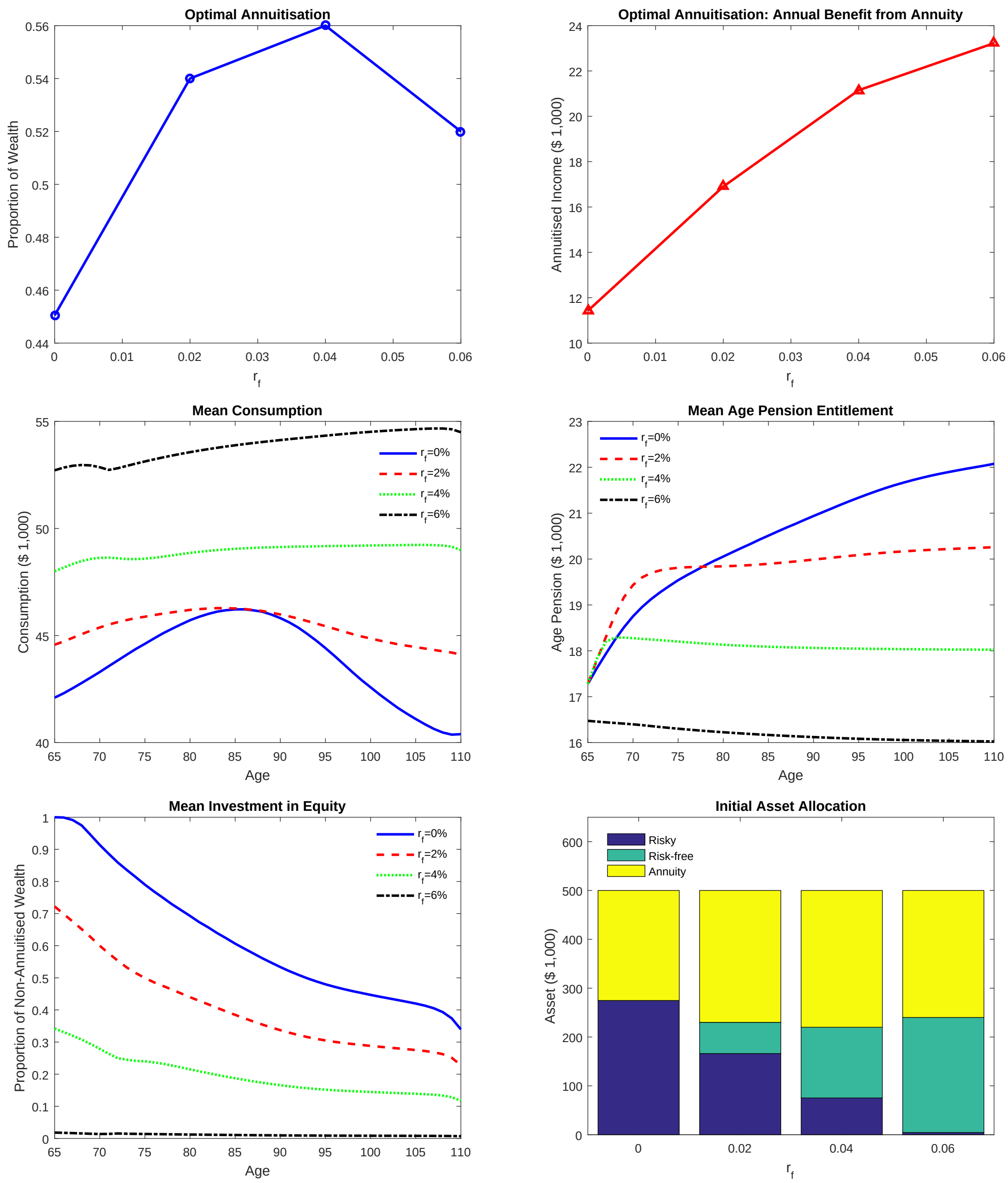


Figure 9.2: Sensitivity analysis of risk-free interest rate on the optimal annuitisation ratio (top left panel), the optimal annual benefit from purchased annuity (top right panel), the optimal mean consumption path (middle left panel), the mean Age Pension entitlements received (middle right panel), the optimal mean proportion of non-annuitised wealth invested in equity (bottom left panel), and the asset allocation at retirement (bottom right panel).

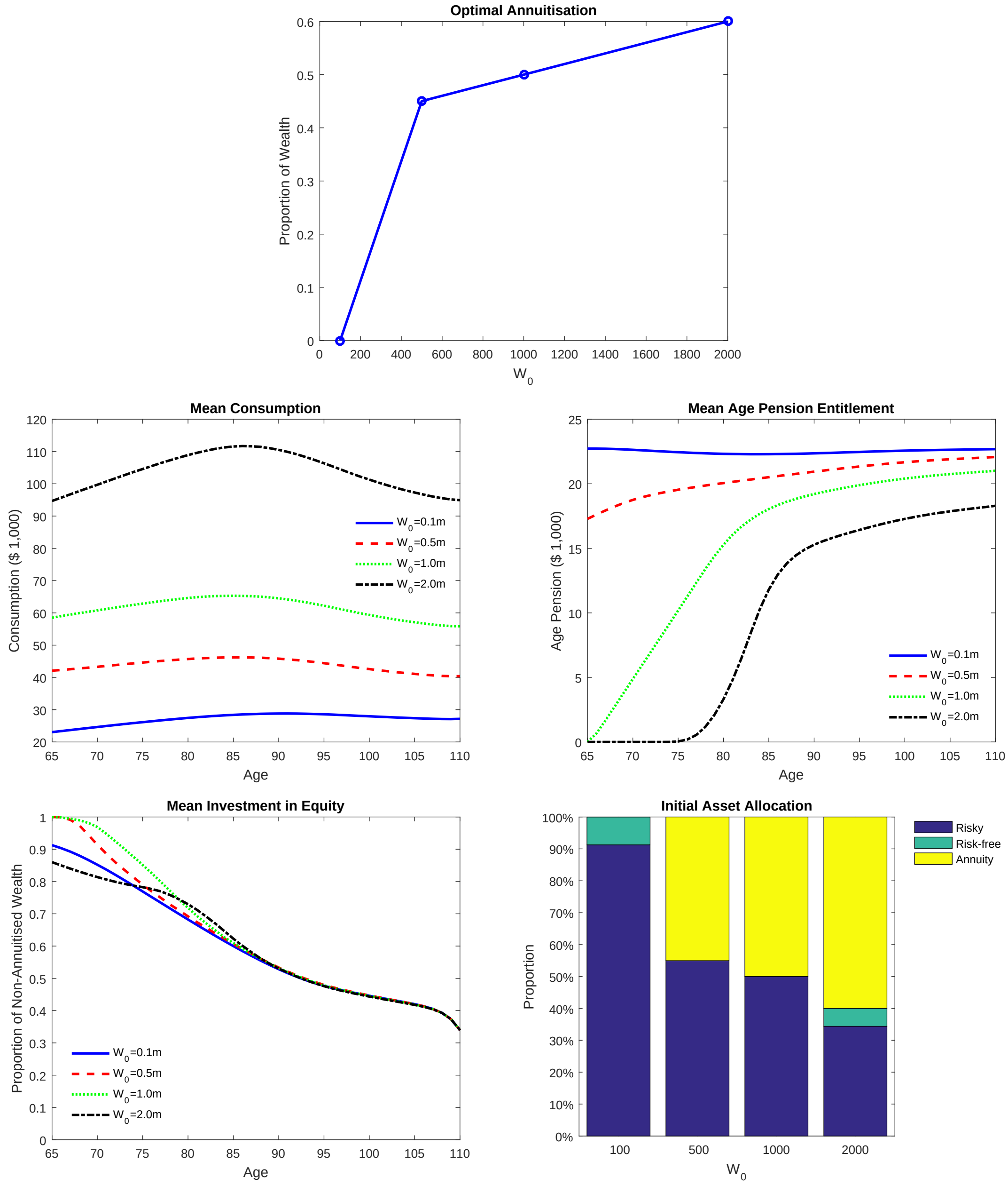


Figure 9.3: Sensitivity analysis of the initial wealth on the optimal annuitisation ratio (top panel), the optimal mean consumption path (middle left panel), the mean Age Pension entitlements received (middle right panel), the optimal mean proportion of non-annuitised wealth invested in equity (bottom left panel), and the asset allocation at retirement (bottom right panel).

9.1.4 Residual Benefit Motive Strength

The impact of residual benefit motive strength parameter is shown in Figure 9.4, from which we see that a higher residual benefit motive generally reduces the level of optimal allocation to equity. The optimal annuitisation ratio is largely reduced when the retiree has a very high residual benefit motive. For members without any residual benefit motive, i.e. $\phi = 0$, the optimal annuitisation ratio is 70% and the optimal expected investment strategy is to invest all non-annuitised assets to equity. The ending point for the blue line in the bottom left panel (i.e. for $\phi = 0$ in the mean investment strategy plot) becomes 0 or Nil, because the optimal strategy is to consume all assets in the last period for members without any residual benefit motive. This also results in the sharp increase in the expected consumption path in the terminal period. We also observe that the generally bell shape of expected consumption with respect to age is averted when the member has a very large residual benefit motive and thus his/her consumption level becomes very low in early years.

9.1.5 Equity Return Distributions

We then investigate the sensitivity of our results on equity return distributions. The expected return is varied from as low as -1% p.a. to as high as 8% p.a. The results are shown in Figure 9.5. The impact of standard deviation of equity returns is shown in Figure 9.6. As expected, the optimal allocation to equity is very sensitive to the trustee's estimation of the distribution of risky assets. The optimal annuitisation increases when equity returns have a lower mean value and a higher volatility, as lower mean and higher volatility both decreases the opportunity cost of annuitisation.

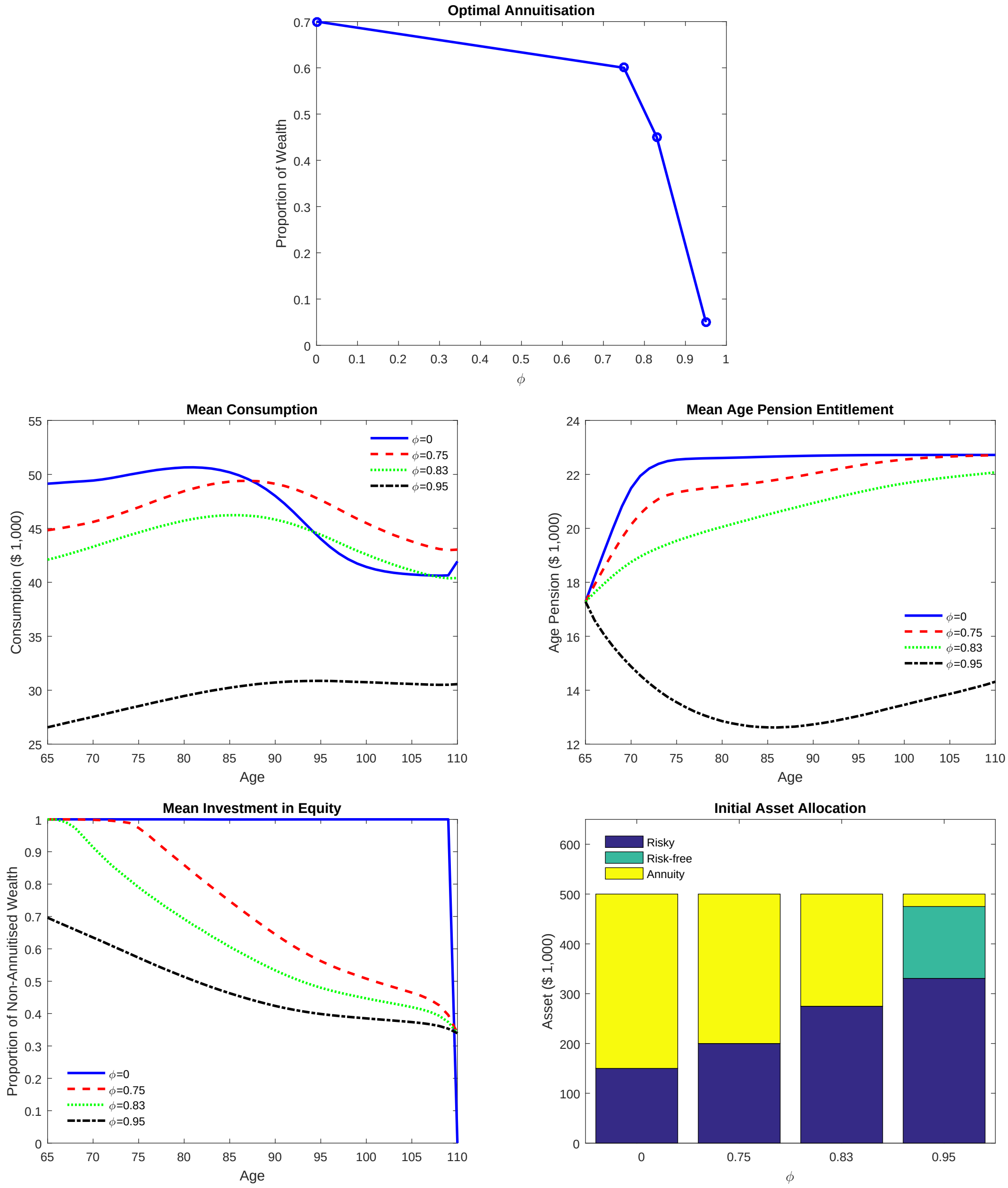


Figure 9.4: Sensitivity analysis of the residual benefit motive strength on the optimal annuitisation ratio (top panel), the optimal mean consumption path (middle left panel), the mean Age Pension entitlements received (middle right panel), the optimal mean proportion of non-annuitised wealth invested in equity (bottom left panel), and the asset allocation at retirement (bottom right panel).

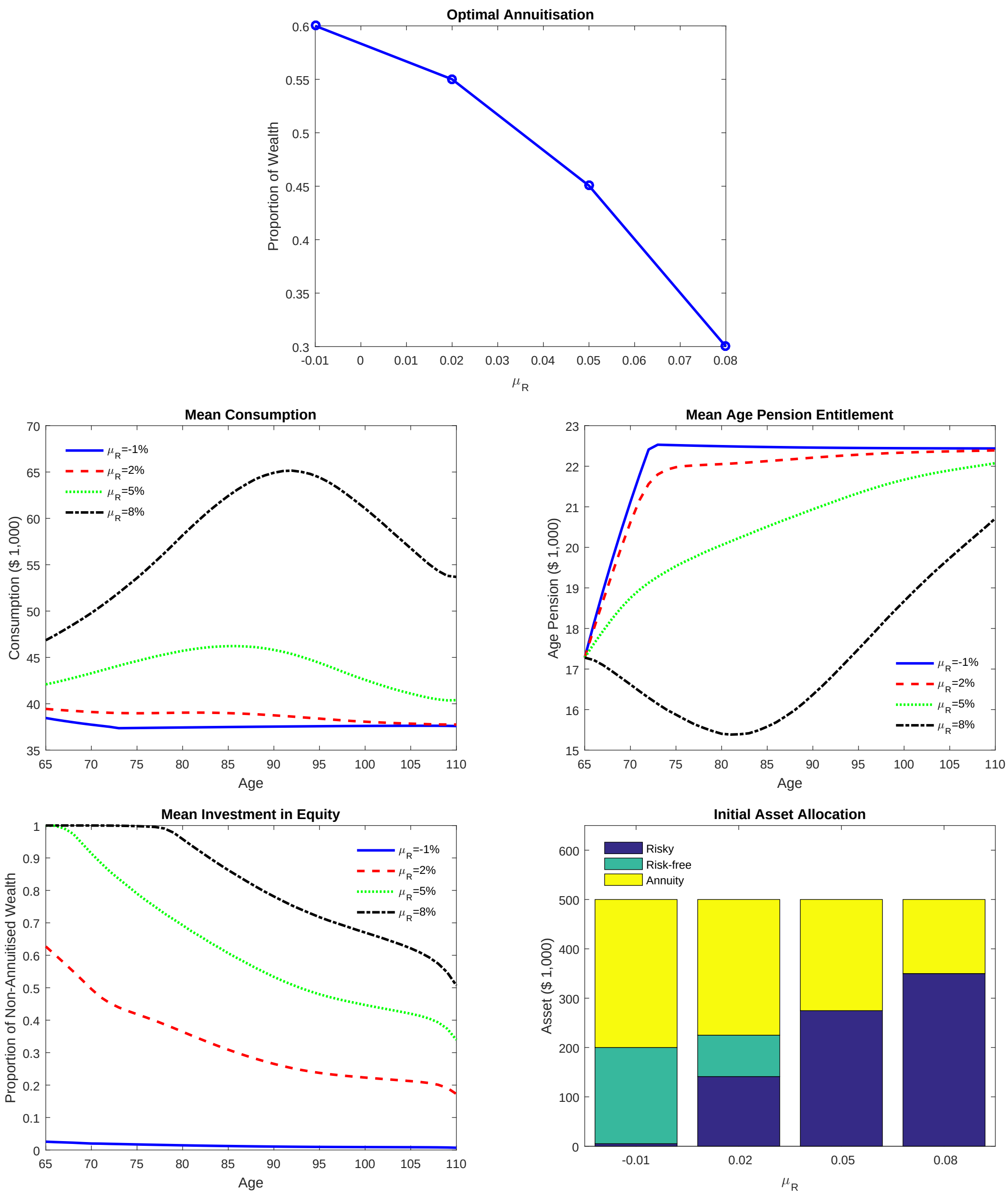


Figure 9.5: Sensitivity analysis of equity return's mean on the optimal annuitisation ratio (top panel), the optimal mean consumption path (middle left panel), the mean Age Pension entitlements received (middle right panel), the optimal mean proportion of non-annuitised wealth invested in equity (bottom left panel), and the asset allocation at retirement (bottom right panel).

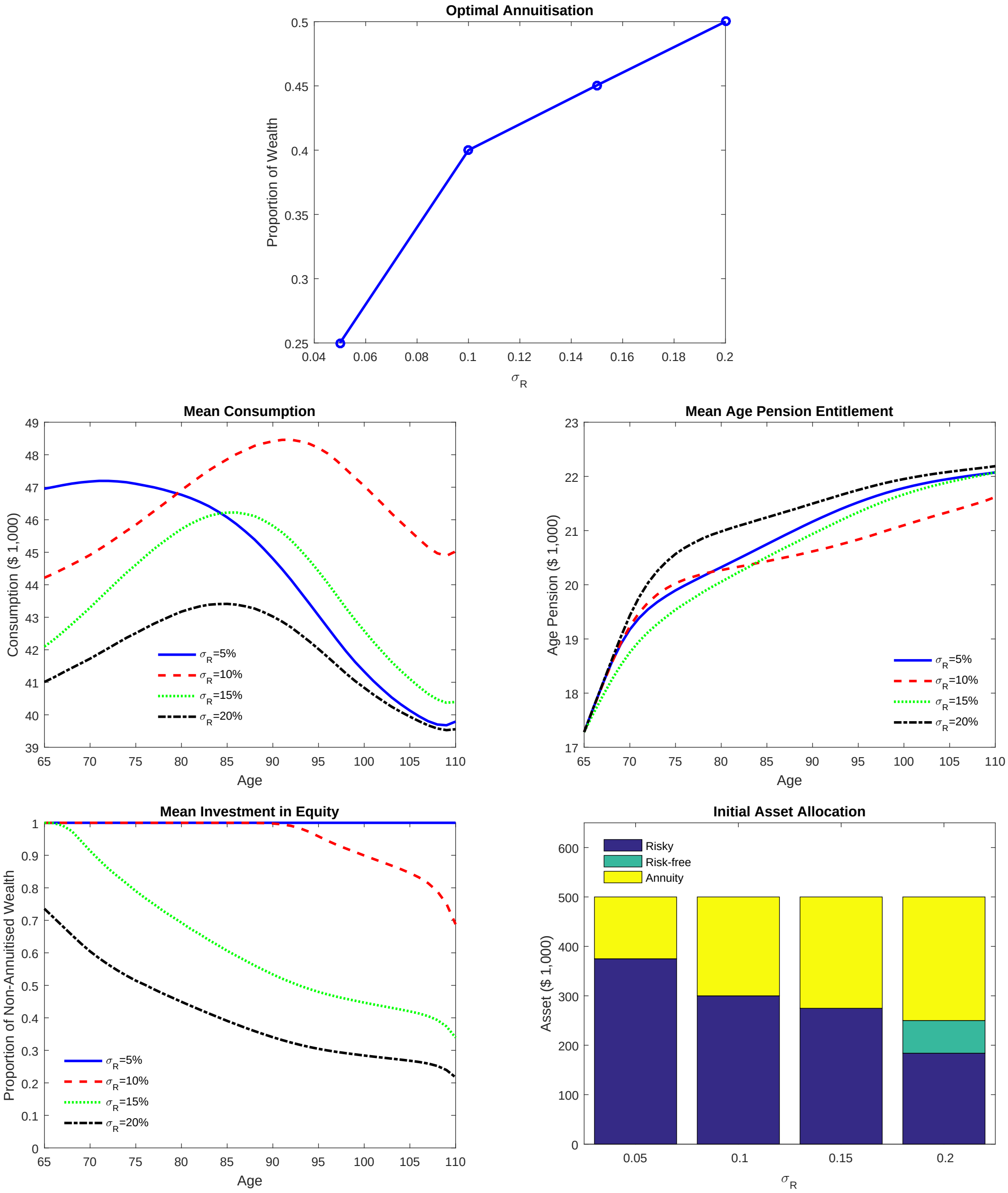


Figure 9.6: Sensitivity analysis of equity return's standard deviation on the optimal annuitisation ratio (top panel), the optimal mean consumption path (middle left panel), the mean Age Pension entitlements received (middle right panel), the optimal mean proportion of non-annuitised wealth invested in equity (bottom left panel), and the asset allocation at retirement (bottom right panel).

9.2 More Explanation on the Impact of Residual Benefit Motive

9.2.1 A Simple Model

We illustrate the impact of residual benefit motive on consumption using an N-period simple example where individuals know when they will die and plan to consume a fixed level of consumption throughout their life.

The asset growth rate is assumed to be 0 in this simple example. The utility maximisation objective function can be written as:

$$\begin{aligned} & \max_{\{c\}} \left\{ N\ u(c) + v(b) \right\} \\ = & \max_{\{c\}} \left\{ N \frac{c^{1-\rho}}{1-\rho} + \left(\frac{\phi}{1-\phi} \right)^{\rho} \frac{b^{1-\rho}}{1-\rho} \right\}, \end{aligned} \tag{9.1}$$

where c is the streamlined consumption level and b is the residual benefit at the pre-determined end of horizon. Given an initial wealth of b_0, the wealth dynamics can be written as

$$b_0 = cN + b. \tag{9.2}$$

The optimal streamlined consumption is obtained by setting the first-order derivative of Equation (9.1) to 0, i.e.

$$\begin{aligned} 0 &= \frac{\partial N\ u(c)}{\partial c} + \frac{\partial v(b)}{\partial c} \\ &= \frac{N}{c^{\rho}} - N \left(\frac{\phi}{1-\phi} \right)^{\rho} \frac{1}{b^{\rho}}. \end{aligned} \tag{9.3}$$

By re-arranging the above equation, we obtain the following relationship between the total consumption vs. residual benefit:

$$\frac{Nc}{b} = \frac{N(1-\phi)}{\phi}. \tag{9.4}$$

In our base case analysis where the residual benefit motive strength parameter, i.e. ϕ, is equal to 0.83. With a 20 years' horizon, the ratio of total consumption to residual benefit is 4.10, implying that we are valuing much more on consumption than residual benefit.

Below is a figure that shows how the ratio of total consumption to residual benefit changes with respect to different residual benefit motive strength levels (ϕ) and horizon lengths (N). We see that the ratio of total consumption to residual benefit increases as the time horizon is longer and the residual benefit motive strength is lower. For a very extreme case where residual benefit motive strength of 1, which means that the individual only values the amount in the residual benefit, the ratio of total consumption to residual benefit is constant at 0. For a residual benefit motive strength of around 0.7, we see that the ratio of total consumption to residual benefit can be as high as 16 when the time horizon is 30 years. At a residual benefit motive strength of 0.5, the ratio of total consumption to residual benefit can be as high as 30 if the individual's horizon is 30 years.

9.2.2 Taking into Account an Approximation of the Age Pension

We use a linear approximation of the Age Pension, i.e. to model means-tested Age Pension entitlement as a linear function with respect to asset level. The calculation of the approximated

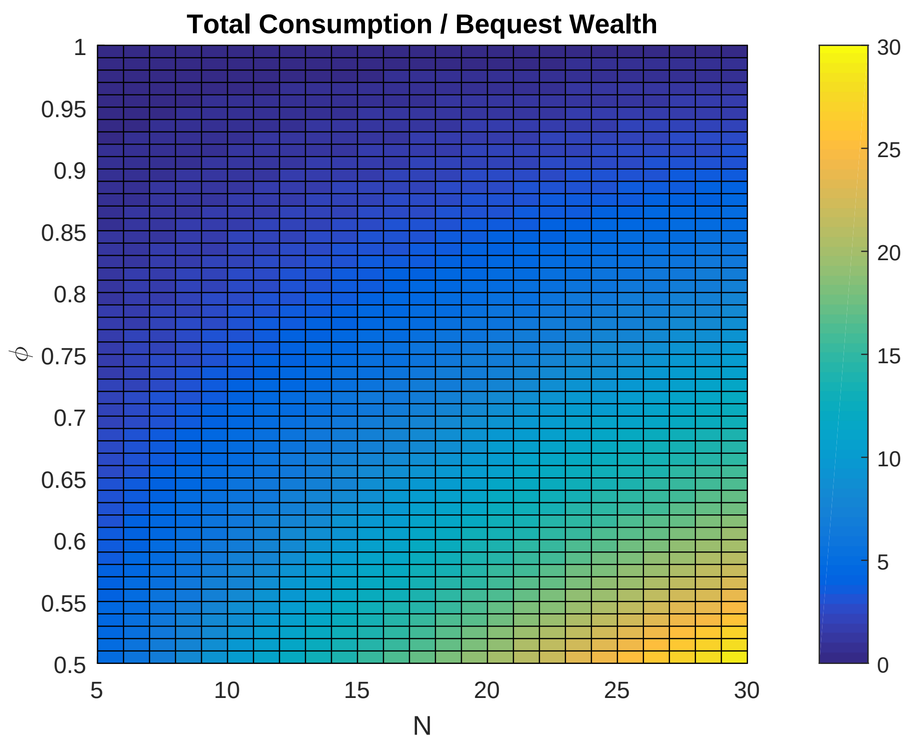


Figure 9.7: Ratio of total consumption to residual benefit for different residual benefit motive and horizon lengths, without taking into account the Age Pension.

Age Pension entitlement is:

$$AP_t = \alpha - \tau b_t, \tag{9.5}$$

where AP_t is the approximated Age Pension entitlement for year t, α is the full Age Pension rate, and τ is the approximated Age Pension taper rate that in our case ensures a non-negative approximated Age Pension entitlement.

In the simple framework in this Appendix, the wealth dynamics is as follows:

$$b_t = b_0 - tc + \sum_{k=0}^{t-1} AP_k. \tag{9.6}$$

Starting from the first period, the amount of approximated Age Pension entitlement in this simple model is:

$$\begin{aligned} AP_0 &= \alpha - \tau b_0 \\ AP_1 &= \alpha - \tau\Big(b_0 - c + AP_0\Big) \\ AP_2 &= \alpha - \tau\Big(b_0 - 2c + (AP_0 + AP_1)\Big) \\ &\cdots \\ AP_t &= \alpha - \tau\Big(b_0 - tc + \sum_{k=0}^{t-1} AP_k\Big). \end{aligned} \tag{9.7}$$

The amount of residual benefit at the end of life in this framework is

$$b = b_0 - Nc + \sum_{k=0}^{N-1} AP_k. \tag{9.8}$$

The partial derivative of the approximated Age Pension entitlement for each period with respect to consumption level is:

$$\begin{aligned}
\frac{\partial}{\partial c}AP_0 &= 0 \\
\frac{\partial}{\partial c}AP_1 &= -\tau\Big(-1+\frac{\partial}{\partial c}AP_0\Big) \;=\; \tau \\
\frac{\partial}{\partial c}AP_2 &= -\tau\Big(-2+\frac{\partial}{\partial c}\left(AP_0+AP_1\right)\Big) \;=\; \tau\left(2-\tau\right) \\
&\cdots \\
\frac{\partial}{\partial c}AP_t &= -\tau\Big(-t+\sum_{k=0}^{t-1}\frac{\partial}{\partial c}AP_k\Big) \;=\; 1-\Big(1-\tau\Big)^t.
\end{aligned} \tag{9.9}$$

Therefore, the partial derivative of residual benefit with respect to consumption is:

$$\begin{aligned}
\frac{\partial b}{\partial c} &= -N+\sum_{k=0}^{N-1}\frac{\partial}{\partial c}AP_k \\
&= -N+\sum_{k=0}^{N-1}\Big[1-\Big(1-\tau\Big)^k\Big] \\
&= -\sum_{k=0}^{N-1}\Big[\Big(1-\tau\Big)^k\Big] \\
&= \begin{cases} -\frac{1-(1-\tau)^N}{\tau} & \tau\neq 0 \\ -N & \tau=0. \end{cases}
\end{aligned} \tag{9.10}$$

The optimal streamlined consumption is then obtained via the following equation:

$$\begin{aligned}
0 &= \frac{\partial N\ u(c)}{\partial c}+\frac{\partial v(b)}{\partial c} \\
&= \frac{N}{c^\rho}+\frac{\partial b}{\partial c}\left(\frac{\phi}{1-\phi}\right)^\rho\frac{1}{b^\rho} \\
&= \begin{cases} \frac{N}{c^\rho}-\frac{1-(1-\tau)^N}{\tau}\left(\frac{\phi}{1-\phi}\right)^\rho\frac{1}{b^\rho} & \tau\neq 0 \\ \frac{N}{c^\rho}-N\left(\frac{\phi}{1-\phi}\right)^\rho\frac{1}{b^\rho} & \tau=0. \end{cases}
\end{aligned} \tag{9.11}$$

Solving the above equation, we obtain the optimal ratio of total consumption to residual benefit as follows:

$$\frac{Nc}{b}=\frac{N(1-\phi)}{\phi}\left(\frac{\tau N}{1-(1-\tau)^N}\right)^{1/\rho}. \tag{9.12}$$

Figure 9.8 shows how the ratio of total consumption to residual benefit changes with respect to the taper ratio and residual benefit motive strength where the time horizon is fixed at 20 years and the risk aversion parameter is equal to 8. We see that the ratio of total consumption to residual benefit increases as the approximated Age Pension taper rate increases and has got nothing to do with the full Age Pension rate. The impact of the Age Pension, however, is not as large as the residual benefit motive.

Figure 9.8: Ratio of total consumption to residual benefit for different residual benefit motive and approximated Age Pension taper rate, assuming a fixed 20 years' horizon and a risk aversion level of 8 (i.e. $\rho = 8$).

9.3 More Explanation on the Risk Aversion Parameter

We use a simple example to illustrate an intuitive interpretation of the risk aversion parameter. Suppose we are faced with two assets: risk-free asset, of which the return is deterministic at r_f; and a risky asset whose return follows a Normal distribution with mean μ and standard deviation σ.

One question arises: At a given risk aversion level, what combinations of μ and σ would give the same utility as the risk-free asset would do?

The answer is: $\mu + (1 - \rho)/2\sigma^2 = r_f$, which gives the risk-return trade-off faced by individuals with a risk aversion parameter rho. The larger the risk aversion parameter, the more penalty individuals would put on volatility.

In our base case analysis the risk aversion parameter is equal to 8. The risk-return trade-off valuation becomes $\mu - 3.5\sigma^2$. This implies that individuals view assets that offer 1 basis point of extra returns the same as those that offer 0.29 basis point reduction in variance (or 0.53 basis point reduction in volatility).

Figure 9.9 shows the indifference curves at varying risk aversion parameters, from which we can see the trade-off between risk and return for a given risk aversion parameter. The top left part of each indifference curve is more preferred than those on the curve or below. We can see that at the same level of mean of risk premium, the corresponding volatility is larger as the risk aversion parameter is small, implying a higher level of risk tolerance.

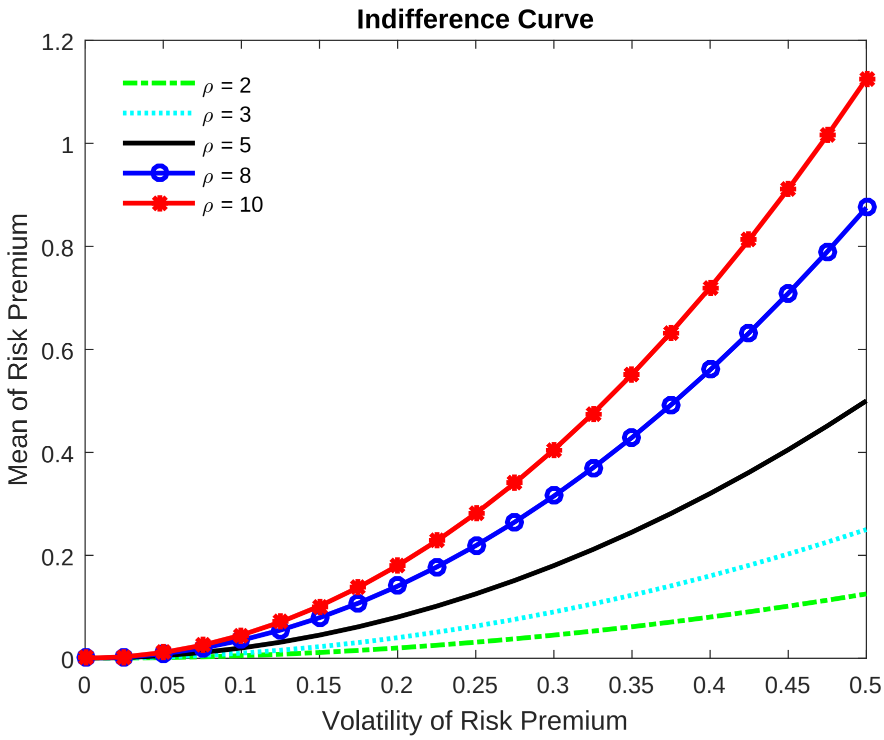


Figure 9.9: Risk-return indifference curves for different risk aversion parameters.

References

Ameriks, J., Caplin, A., Laufer, S. and Van Nieuwerburgh, S. (2011), 'The joy of giving or assisted living? using strategic surveys to separate public care aversion from bequest motives', *Journal of Finance* **66**(2), 519–561.

Asher, A. (2007), Smoothing investment returns, *in* H. Bateman, ed., 'Retirement Provision in Scary Markets', Edward Elgar Publishing, chapter 8.

Bajtelsmit, V. L. and Bernasek, A. (2001), *Risk preferences and the investment decisions of older Americans*, AARP, Public Policy Institute.

Battocchio, P. and Menoncin, F. (2004), 'Optimal pension management in a stochastic framework', *Insurance: Mathematics and Economics* **34**(1), 79–95.

Becker, G. S. and Murphy, K. M. (1988), 'A theory of rational addiction', *Journal of Political Economy* pp. 675–700.

Bengen, W. P. (2004), 'Determining withdrawal rates using historical data', *Journal of Financial Planning* **17**(3), 64–73.

Blake, D., Wright, D. and Zhang, Y. (2014), 'Age-dependent investing: Optimal funding and investment strategies in defined contribution pension plans when members are rational life cycle financial planners', *Journal of Economic Dynamics and Control* **38**, 105–124.

Blanchett, D., Finke, M. and Guillemette, M. (2014), Variable risk preference bias, Technical report, Morningstar.

Boulier, J.-F., Huang, S. and Taillard, G. (2001), 'Optimal management under stochastic interest rates: the case of a protected defined contribution pension fund', *Insurance: Mathematics and Economics* **28**(2), 173–189.

Broome, J. (1991), 'Weighing goods: Equality, uncertainty and time'.

Brown, J. R. (2001), 'Private pensions, mortality risk, and the decision to annuitize', *Journal of Public Economics* **82**(1), 29–62.

Cairns, A. J., Blake, D. and Dowd, K. (2006), 'Stochastic lifestyling: Optimal dynamic asset allocation for defined contribution pension plans', *Journal of Economic Dynamics and Control* **30**(5), 843–877.

Carroll, C. D. (2000), Portfolios of the rich, Technical report, National bureau of economic research.

Chetty, R. (2006), 'A new method of estimating risk aversion', *The American Economic Review* pp. 1821–1834.

Cocco, J. F., Gomes, F. J. and Maenhout, P. J. (2005), 'Consumption and portfolio choice over the life cycle', *Review of Financial Studies* **18**(2), 491–533.

Constantinides, G. M. (1990), 'Habit formation: A resolution of the equity premium puzzle', *Journal of Political Economy* pp. 519–543.

Deaton, A. and Muellbauer, J. (1980), 'An almost ideal demand system', *The American Economic Review* pp. 312–326.

Deelstra, G., Grasselli, M. and Koehl, P.-F. (2003), 'Optimal investment strategies in the presence of a minimum guarantee', *Insurance: Mathematics and Economics* **33**(1), 189–207.

Devolder, P., Princep, M. B. and Fabian, I. D. (2003), 'Stochastic optimal control of annuity contracts', *Insurance: Mathematics and Economics* **33**(2), 227–238.

Di Giacinto, M., Federico, S. and Gozzi, F. (2011), 'Pension funds with a minimum guarantee: a stochastic control approach', *Finance and Stochastics* **15**(2), 297–342.

Ding, J., Kingston, G. and Purcal, S. (2014), 'Dynamic asset allocation when bequests are luxury goods', *Journal of Economic Dynamics and Control* **38**, 65–71.

Elster, J. (1986), *Rational choice*, NYU Press.

Epstein, L. G. and Zin, S. E. (1989), 'Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework', *Econometrica: Journal of the Econometric Society* pp. 937–969.

Ferson, W. E. and Constantinides, G. M. (1991), 'Habit persistence and durability in aggregate consumption: Empirical tests', *Journal of Financial Economics* **29**(2), 199–240.

Frederick, S. (1999), Discounting, time preference, and identity, PhD thesis, Carnegie Mellon University.

Frederick, S., Loewenstein, G. and O'donoghue, T. (2002), 'Time discounting and time preference: A critical review', *Journal of Economic Literature* **40**(2), 351–401.

Friend, I. and Blume, M. E. (1975), 'The demand for risky assets', *The American Economic Review* pp. 900–922.

Ganegoda, A. and Bateman, H. (2008), 'Australia's disappearing market for life annuities', *UNSW Centre for Pensions and Superannuation Discussion Paper* **1**(08).

Gao, J. (2008), 'Stochastic optimal control of dc pension funds', *Insurance: Mathematics and Economics* **42**(3), 1159–1164.

Gourinchas, P.-O. and Parker, J. A. (2002), 'Consumption over the life cycle', *Econometrica* **70**(1), 47–89.

Haberman, S. and Vigna, E. (2002), 'Optimal investment strategies and risk measures in defined contribution pension schemes', *Insurance: Mathematics and Economics* **31**(1), 35–69.

Hall, R. E. (1978), 'Stochastic implications of the life cycle-permanent income hypothesis: Theory and evidence', *Journal of Political Economy* **86**(6), 971–987.

Hansen, L. P. and Jagannathan, R. (1991), 'Restrictions on intertemporal marginal rates of substitution implied by asset returns', *Journal of Political Economy* **99**(2), 225.

Horneff, W. J., Maurer, R. H., Mitchell, O. S. and Stamos, M. Z. (2009), 'Asset allocation and location over the life cycle with investment-linked survival-contingent payouts', *Journal of Banking & Finance* **33**(9), 1688–1699.

Horneff, W. J., Maurer, R. H. and Stamos, M. Z. (2008), 'Life-cycle asset allocation with annuity markets', *Journal of Economic Dynamics and Control* **32**(11), 3590–3612.

Janecek, K. (2002), What is a realistic aversion to risk for real-world individual investors, Technical report, Citeseer.

Kahneman, D. and Tversky, A. (1979), 'Prospect theory: An analysis of decision under risk', *Econometrica: Journal of the Econometric Society* pp. 263–291.

Kingston, G. and Thorp, S. (2005), 'Annuitization and asset allocation with hara utility', *Journal of Pension Economics and Finance* **4**(03), 225–248.

Lockwood, L. M. (2014), Incidental bequests: Bequest motives and the choice to self-insure late-life risks, Technical report, National Bureau of Economic Research.

Markowitz, H. (1952), 'Portfolio selection', *Journal of Finance* **7**(1), 77–91.

Mehra, R. and Prescott, E. C. (1985), 'The equity premium: A puzzle', *Journal of Monetary Economics* **15**(2), 145–161.

Meyer, D. J. and Meyer, J. (2005), 'Relative risk aversion: What do we know?', *Journal of Risk and Uncertainty* **31**(3), 243–262.

Modigliani, F. and Brumberg, R. (1954), 'Utility analysis and the consumption function: An interpretation of cross-section data', *Post-Keynesian Economics* **1**.

Montserrat, G., Jørgensen, P. L. and Nielsen, J. P. (2006), 'Return smoothing mechanisms in life and pension insurance', *Insurance: Mathematics and Economics* .

Morin, R.-A. and Suarez, A. F. (1983), 'Risk aversion revisited', *Journal of Finance* pp. 1201–1216.

Parfit, D. (1993), 'The indeterminacy of identity: A reply to brueckner', *Philosophical Studies* **70**(1), 23–33.

Rawls, J. (2009), *A theory of justice*, Harvard university press.

Schechter, L. (2007), 'Risk aversion and expected-utility theory: A calibration exercise', *Journal of Risk and Uncertainty* **35**(1), 67–76.

Starmer, C. (2000), 'Developments in non-expected utility theory: The hunt for a descriptive theory of choice under risk', *Journal of Economic Literature* pp. 332–382.

Tobin, J. (1958), 'Liquidity preference as behavior towards risk', *The Review of Economic Studies* pp. 65–86.

Tobin, J. and Dolde, W. (1971), Wealth, liquidity and consumption, *in* 'Consumer spending and monetary policy: The linkages', Vol. 5, Federal Reserve Bank of Boston Boston, pp. 99–146.

Wang, H. and Hanna, S. D. (1997), 'Does risk tolerance decrease with age?', *Financial Counseling and Planning* **8**(2).

Xiao, J., Hong, Z. and Qin, C. (2007), 'The constant elasticity of variance (cev) model and the legendre transform–dual solution for annuity contracts', *Insurance: Mathematics and Economics* **40**(2), 302–310.

Yogo, M. (2009), Portfolio choice in retirement: Health risk and the demand for annuities, housing, and risky assets, Technical report, National Bureau of Economic Research.

Zemach, E. M. (1987), 'Looking out for number one', *Philosophy and Phenomenological Research* **48**(2), 209–233.