# Member's Default Utility Function Version 1 ("MDUF v1")

# Technical Note - Static Model Calculator

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December 7, 2017

## 1 Introduction

This paper provides technical explanation of the measures and calculation in the excel static model calculator. It should be read in conjunction with the excel model.

## 2 MDUF Measures

This section details the different measures we used in comparing and ranking static retirement solutions. These measures include expected utility, MDUF score, welfare gain and risk-adjusted income and residual benefit.

### 2.1 Expected Utility

Expected utility ($U_0$) can be used as a scoreboard of an individual's lifetime welfare, which is a ranking instead of an absolute score. It is expressed in the following formulae:

$$\boxed{U_0 = \mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \left\{ {}_tp_x \, \frac{c_t^{1-\rho}}{1-\rho} + {}_{t-1|}q_x \, \frac{b_t^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho} \right\}\right]}$$

- $x$: the inception age of a particular cohort,
- $T$: the retirement planning horizon ($x + T$ is the maximum age),
- $c_t$: consumption/income in year $t$[1],

We would like to acknowledge the significant contribution from Dr. Adam Shao as a lead author of MDUF v1 when he was at Mine Wealth and Wellbeing. He is now at Milliman and he is also an Associate Investigator at CEPAR (ARC Centre of Excellence in Population Ageing Research).

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[1]The terms "consumption" and "income" are used interchangeably through all the MDUF research materials including this document

- $b_t$: level of wealth at time $t$ which equals the amount of residual benefit if the person dies between $t-1$ and $t$,
- ${}_tp_x$: the probability of being alive at age $x+t$ conditional on being alive at age $x$ (Mortality rates can be sourced from the Australian Life Tables 2010-12 by Australian Government Actuary[2]),
- ${}_{t-1|}q_x$: the probability of dying between age $x+t-1$ and $x+t$ conditional on being alive at age $x$ (${}_{t-1|}q_x$ can be calculated from ${}_tp_x$),
- $\beta = 1$: subjective utility discount factor that captures the person's time preference for near versus far-dated income and residual benefit,
- $\rho = 8$: level of risk aversion, and
- $\phi = 0.83$: strength of residual benefit motive.

## 2.2 MDUF Score

It is not easy to perform quantitative comparisons directly based on utility scores, therefore other measures such as MDUF Score ($S_0$) becomes very useful. MDUF Score is a monotonic transformation of the expected utility. It is the constant level of income (considering the trade-off against residual benefit) which delivers an equivalent level of expected utility. It is calculated based on the formulae below:

$$\boxed{S_0 = \left[U_0 \times \frac{1-\rho}{\left[\sum_{t=0}^{T} \beta^t \{{}_tp_x + {}_{t-1|}q_x \frac{\phi}{1-\phi}\}\right]}\right]^{\frac{1}{1-\rho}}}$$

When $\beta = 1$, $\sum_{t=0}^{T} {}_tp_x$ can be considered as the life expectancy of the person at age $x$ and $\sum_{t=0}^{T} {}_{t-1|}q_x = 1$. To derive $S_0$, we firstly assume time independent income $c_t = S_0$ and residual benefit $b_t = B_0$ for all $t$ in the expected utility formulae and equate it to $U_0$. This gives

$$U_0 = \mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \left\{ {}_tp_x \, \frac{S_0^{1-\rho}}{1-\rho} + {}_{t-1|}q_x \, \frac{B_0^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho} \right\}\right] \tag{2.1}$$

Recognising the trade-off between income and residual benefit by $B_0 = S_0 \frac{\phi}{1-\phi}$ gives

$$\begin{aligned} U_0 &= \mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \left\{ {}_tp_x \, \frac{S_0^{1-\rho}}{1-\rho} + {}_{t-1|}q_x \, \frac{(S_0 \frac{\phi}{1-\phi})^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right)^{\rho} \right\}\right] \\ &= \sum_{t=0}^{T} \beta^t \left\{ {}_tp_x \, \frac{S_0^{1-\rho}}{1-\rho} + {}_{t-1|}q_x \, \frac{S_0^{1-\rho}}{1-\rho}\left(\frac{\phi}{1-\phi}\right) \right\} \end{aligned} \tag{2.2}$$

Rearranging the equation would give you the formulae for $S_0$.

## 2.3 Welfare Gain

For a pair of cases, welfare gain is the additional initial wealth required for a inferior solution to achieve the same expected utility as a superior solution. We can compare cases with and

[2] The Australian Life Tables 2010-12 can be downloaded via `http://www.aga.gov.au/publications/life_table_2010-12/`

without a certain retirement product (e.g. life annuity) in order to measure the dollar amount of welfare gains from having access to this product and using it optimally. We can also use this measure to quantify the value of a better designed retirement strategy.

Welfare gain ($G_0$) is calculated based on the expected present value of income and residual benefit. It is estimated through MDUF score using the formulae below:

$$\boxed{G_0 = (S_0 - S_0^*)\Big[\sum_{t=0}^{T} \beta^t \{{}_tp_x + {}_{t-1|}q_x \frac{\phi}{1-\phi}\}\Big]}$$

Where $S_0^*$ is the MDUF score of the base case that this solution is comparing against. MDUF score ($S_0$) is considered as a proxy of income, the expected present value of incomes is then:

$$S_0 \sum_{t=0}^{T} \beta^t {}_tp_x \tag{2.3}$$

The expected present value of residual benefit is:

$$\begin{aligned} &B_0 \sum_{t=0}^{T} \beta^t {}_{t-1|}q_x \\ =&S_0 \frac{\phi}{1-\phi} \sum_{t=0}^{T} \beta^t {}_{t-1|}q_x \end{aligned} \tag{2.4}$$

Sum up 2.3 and 2.4 would result in the expected present value of the two components. As a result, subtracting $S_0^*$ from $S_0$ in the formulae would give the welfare gain of this solution over the base case solution.

## 2.4 Risk-Adjusted Income

Risk-adjusted income ($S_c$) is the constant level of income which delivers an equivalent level of income utility. Income utility is the expected utility with the residual benefit component set to zero. This measure focuses on the income component only. It is useful when we want to separate the two components income and residual benefit and see their impacts on the overall utility. It is calculated using the formulae below:

$$\boxed{S_c = \left[U_c \times \frac{1-\rho}{\sum_{t=0}^{T} \beta^t {}_tp_x}\right]^{\frac{1}{1-\rho}}}$$

Where $U_c$ is the portion of the expected utility that comes from the income component only. It is calculated using the formulae below:

$$U_c = \mathbb{E}_0\Big[\sum_{t=0}^{T} \beta^t {}_tp_x \frac{c_t^{1-\rho}}{1-\rho}\Big] \tag{2.5}$$

To derive $S_c$, we assume time independent income $c_t = S_c$ in formulae 2.5 for all $t$ and equate it to $U_c$. This gives

$$U_c = \mathbb{E}_0\Big[\sum_{t=0}^{T} \beta^t {}_tp_x \frac{S_c^{1-\rho}}{1-\rho}\Big] \tag{2.6}$$

Rearranging the equation would give you the formulae for $S_c$.

### 2.5 Risk-Adjusted Residual Benefit

Risk-adjusted residual benefit ($S_b$) is the constant level of residual benefit which delivers an equivalent level of residual benefit utility. Residual benefit utility is the expected utility with the income component set to zero. This measure focuses on the residual benefit component only. Similar to the risk-adjusted income measure, this measure is useful when we want to separate the two components income and residual benefit and see their impacts on the overall utility. It is calculated using the formulae below:

$$\boxed{S_b = \left[U_b \times \frac{1-\rho}{\sum_{t=0}^{T} \beta^t {}_{t-1|}q_x (\frac{\phi}{1-\phi})^\rho}\right]^{\frac{1}{1-\rho}}}$$

Where $U_b$ is the portion of the expected utility that comes from the residual benefit component only. It is calculated using the formulae below:

$$U_b = \mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \left\{ {}_{t-1|}q_x \, \frac{b_t^{1-\rho}}{1-\rho} \left(\frac{\phi}{1-\phi}\right)^\rho \right\}\right] \tag{2.7}$$

To derive $S_b$, we assume time independent residual benefit $b_t = S_b$ in formulae 2.7 for all $t$ and equate it to $U_b$. This gives

$$U_b = \mathbb{E}_0\left[\sum_{t=0}^{T} \beta^t \left\{ {}_{t-1|}q_x \, \frac{S_b^{1-\rho}}{1-\rho} \left(\frac{\phi}{1-\phi}\right)^\rho \right\}\right] \tag{2.8}$$

Rearranging the equation would give you the formulae for $S_b$.

## 3 Floors on Income and Residual Benefit

Due to the nature of the means tested Age Pension, there is an implicit floor on incomes. Incomes would never drop below the Age Pension entitlement level. This is applied consistently across to residual benefit. Otherwise, static strategies are likely to be penalised heavily when there is even only a very small probability of running out of savings. As a result, an explicit floor which equals to the Age Pension entitlement of the person at that time is added. This is only a consideration in comparing static solutions, dynamic solutions would not require explicit floors for residual benefit as optimal solution would not be in that solution space.